Numerical modelling of landscape evolution: geomorphological perspectives

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Abstract: A resurgence of interest in landscape evolution has occurred as computational technology has made possible spatially and temporally extended numerical modelling. We review elements of a structured approach to model development and testing. It is argued that natural breaks in landscape process and morphology define appropriate spatial domains for the study of landscape evolution. The concept of virtual velocity is used to define appropriate timescales for the study of landscape change. Process specification in numerical modelling requires that the detail incorporated into equations be commensurable with the particular scale being considered. This may entail a mechanistic approach at small (spatial) scales, whereas a generalized approach to process definition may be preferred in large-scale studies. The distinction is illustrated by parameterizations for hillslope and fluvial transport processes based on scale considerations. Issues relevant to model implementation, including validation, verification, calibration and confirmation, are discussed. Finally, key developments and characteristics associated with three approaches to the study of landscape modelling: (i) conceptual; (ii) quasi-mechanistic; and (iii) generalized physics, are reviewed.

Key words: geomorphology; landscape evolution; model implementation; numerical modelling; process specification; scale.

I Introduction

The ball is now in the geomorphologists’ court. (Anderson and Humphrey, 1989: 350)

This remark appeared in an early discussion of information requirements for modelling the interaction of tectonic and surface processes at large scales. The
adoption of numerical modelling for the study of combined exogene and endogene processes represented a recent development at the time. Researchers were turning to the geomorphological literature to find information about the rate of surface transport processes at large spatial and temporal scales in order to model the lithospheric response to erosion and sedimentation. Anderson and Humphrey found that existing results did not offer the information necessary for successful implementation of such models. Their statement was made at the beginning of a resurgence of interest in landscape evolution that had occurred in recent years (review by Thomas and Summerfield, 1987). In subsequent years, much progress has been made in numerical modelling of landscape evolution (reviews by Merritts and Ellis, 1994; Summerfield, 2000), although many outstanding issues still remain concerning the representation of geomorphological processes over large spatial and temporal scales.

It is proposed herein that numerical modelling of landscape evolution reconciles 'historical' and 'process' studies by explaining characteristic features of historical geomorphological landscapes in terms of physically based processes (without necessarily being able to recreate the details of specific landscapes). Numerical modelling requires a framework within which material fluxes can be reconciled across all resolved scales. The crux of modelling from a geomorphological perspective is how to represent the flux 'laws' governing landforming processes. Adequate calibration and testing of transport relations used in such models remains elusive because of difficulties in acquiring suitable field data over representative timescales. In turn, the litmus test of a model depends on finding appropriately rigorous ways to compare model results with the evidence of particular landscapes. The resolution of both of these problems is dominated by scale. Spatial and temporal scales of a study should guide the specification of process and selection of test procedures within numerical models. A judgement must be made regarding 'flaws' within the model that must be tolerated for the sake of computational tractability, while replicating the essential characteristics of the phenomena under consideration. The dramatically expanded body of tools and techniques now available, including much improved topographic mapping (DEMs), high resolution air photographs, substantially improved surveying instruments and sophisticated dating techniques, have increased our ability to resolve key issues related to process specification and model testing.

Many of the existing landscape models consider the interactions of tectonic and geomorphological processes. At intermediate timescales, geomorphological development of the landscape can be studied independently of tectonic considerations. This approach is increasingly important in studies of 'long-term' effects of landscape management, as well as to study the implications of geomorphological processes over extended periods. An examination of methodology surrounding the numerical representation of geomorphological phenomena at large scales can provide insight into appropriate specifications of processes for a particular scale of study. It can also reveal the weaknesses in our ability to do so. The objective of this paper, then, is to examine theoretical and methodological issues related to the quantitative study of geomorphological processes within the context of landscape evolution.
II Scale

1 Scale and process

Scales are a set of natural measures that are intrinsic to a system. The chosen scale guides the appropriate specification of the system and processes within it. Generally, as the spatial and temporal scales of a study decrease (in the physical, rather than the cartographical, sense) it is possible to resolve greater detail in the processes. Development and calibration of flux relations on the basis of transport rates measured in the field or laboratory is an easier task at smaller scales because the system configuration remains relatively simple. Results obtained in smaller-scale studies cannot be replicated directly at large scales since details found in process equations at smaller scales are no longer resolvable.

A significant dilemma, then, is how to define relations and calibrate them to represent the operation of geomorphological processes at large scales. Consideration must be given to the way in which scale limits the information available to derive a representation of processes. Equations are required that are generalized appropriately to match the information.

To define and calibrate process equations for large scales, emphasis must be placed first on identifying dominant processes at large scales (Murray, 2002), then on establishing appropriate process equations to describe them and, finally, on estimating their rates. To calibrate relations, methods must be developed that yield estimates of long-term transport rates. Advances in radiometric isotope dating (cf. Cerling and Craig, 1994; Ring et al., 1999) and in luminescence dating (Duller, 2000) have occurred that increase the potential to estimate long-term process rates. These methods are proving critical to establishing rates of process operation in landscape evolution (e.g., Bierman, 1994; Garver et al., 1999; Clemmensen et al., 2001; Nichols et al., 2002; amongst many papers). Other approaches that have the potential to provide useful information include the use of remotely sensed imagery, such as aerial photography, to generate large data bases for specific types of geomorphological phenomena, such as shallow landsliding (Guzzetti et al., 2002; Martin et al., 2002) or large failures in bedrock (e.g., Hovius et al., 1997, 2000), and the well-established practice of using digital elevation models to provide a quantitative, easily manipulated record of topography. In short, the potential for improved understanding of process operation at a variety of scales has widened considerably as a result of technological advances.

2 Spatial scale

If scales represent natural measures in a system, then some defining criterion for a natural measure must be selected. One basis for the delineation of a natural measure is to locate natural breaks, often based on process domains, that partition variability in some relevant attribute of the system. Such scalings are often presented (see, for example, Schumm and Lichty, 1965, in geomorphology; Steyn et al., 1981, in climatology; Delcourt et al., 1983, in ecology; Ehleringer and Field, 1993, in plant physiology). In landscape, the primary attribute of concern is morphology. Morphological breaks in the landscape can be used to guide the selection of appropriate domains for the study of landscape change, and the extent of such domains in turn dictates the
scale of study. There are two approaches to identify such breaks. The first is to consider landscape units of traditional geomorphological interest; the second is direct numerical analysis of the landscape to reveal the distribution of variance in key properties. We first explore three alternative choices for the scale of landscape evolution studies that are well-represented in geomorphological textbooks: tectonic units, drainage basins and hillslopes.

The largest customary landscape scale is based on tectonically determined topography (e.g., Summerfield, 1991; Ahnert, 1998). Geology, climate and development history determine the characteristics of a particular tectonic landscape (e.g., Brozovic et al., 1997; Montgomery et al., 2001). The tectonic unit represents the fundamental unit for definition of the balance between uplift and downwearing that exists in evolving landscapes and the largest landscape unit for which there appears to be greater morphological variability between units than within a unit. Studies to understand the evolution of tectonic units constitute the currently most active area of landscape modelling (Beaumont et al., 2000). Most such studies represent attempts to understand generic features of tectonic landscapes, though they may refer to specific landscapes for context or for qualitative comparison. Increasingly, however, investigators (e.g., Koons, 1989; Tucker and Slingerland, 1996; Ellis et al., 1999; Anderson, 2002) are attempting to use numerical models to understand aspects of the history of specific tectonic landscapes.

Drainage basins are spatial units containing integrated areal and linear pathways for sediment movement. The particular characteristics of a drainage basin depend on the magnitude of the basin and on geology and climate. However, all drainage basins by definition exhibit functional similarity insofar as water and sediment are routed through the system to a single outlet. Drainage basins have long been recognized as appropriate study units for hydrological research and analyses of sediment yield (e.g., Chorley, 1969; Chorley et al., 1984). They also form the study unit for research that examines the development of the fluvial system at large scales (e.g., Davis, 1899; Schumm, 1977), hence the focus of studies in which drainage development is investigated (e.g., Smith and Bretherton, 1972; Dunne, 1980; Willgoose et al., 1991a, b; Dietrich et al., 1993). Terrestrial geomorphological processes, with the exception of glacial and aeolian processes (which have not generally been incorporated into landscape models), occur within drainage basin boundaries. The drainage basin is, then, the logical unit within which to model the subaerial geomorphological evolution of landscape (e.g., Tucker and Bras, 2000).

The smallest scale at which landscape evolution can be studied reasonably is the hillslope scale. All hillslopes have a general morphological similarity by definition insofar as they are planar or quasi-planar features bounded by a slope base and a crest at the top. When assembled together, hillslopes constitute the drainage basin surface. This distinctive form indicates a distinctive process unit. Sediment is transported from the upper portions of the hillslope and is deposited along the slope base by processes that, in the long term, are generally diffusive in character. Hillslopes have often been the study unit to consider topographic profile development (Penck, 1953; Culling, 1960; Kirkby, 1971, 1987b; Martin, 2000).

Mark and Aronson (1984) introduced a direct numerical (hence, within the limits of the source data, objective) approach to define topographic scales. They computed the variogram of topography from 30 m digital elevation models (DEMs) of topographic quadrangles with 1-m resolution of elevation. The
variograms (e.g., Figure 1) typically revealed three distinct fractal ranges (implying distinct roughness characteristics), with scale breaks at less than 1 km and less than 10 km. They declared that these breaks indicate important changes in dominant geomorphological process regime, hence, natural scales. The lower scale break evidently marks the limit of the hillslope process regime in the studied landscapes, whilst the upper break marks a limit between the roughness imposed by fluvially organized topography and larger scales imposed by structure. Beyond this break, periodic (rather than fractal) modulation of the landscape is often observed owing to the structural control of relief. Reasons for the breaks have been discussed by other authors – the distinction between diffusive (topographic smoothing) processes on hillslopes, linear (topographic roughening) processes in drainage basins and sedimentation processes (topographic smoothing) at large scales being cited by Culling and Datco (1987) and Chase (1992). Fundamentally, one is observing the different effects of these disparate processes on the growth of topographic variance with changing spatial scale.

Analyses of this type have been surprisingly little pursued. Most analysts who have studied topographic variance have concentrated, rather, on the similarity (self-affinity) of landscapes over a range of scales (review of early work in Xu et al.,

![Figure 1](image_url)

**Figure 1** Relief variograms for field areas within the American Appalachian Mountains. (a) Data for Aughwick, Pennsylvania, quadrangle; arrows indicate significant breaks at the limit of hillslope scales and at the limit of fluvial ridge and valley scales. (b) Trend lines for several field areas, the heavy line is the area shown in (a). *Source: Compiled data from Figure 1a and Figure 4 of Mark and Aronson (1984).*
1993) – an exercise designed to ignore multifractal signatures. In part, as well, artefacts of map digitization may confound results (Xu et al., 1993) and, of course, map scale itself limits what may be observed (Outcalt et al., 1994). Most work has resolved well only the intermediate domain of erosional topography of drainage basins. There remains much work to be done on the partition of topographic variance and the definition of process domains in the landscape.

3 Temporal scale

The selection of spatial and temporal scales for a study cannot be made in isolation. As the spatial domain increases, the detection of a ‘resolvable’ amount of change requires significantly longer periods of observation. Hence, the increase in spatial domain necessitates a change in temporal scale.

Time and length are connected via a measure of velocity: i.e.,

\[
\text{time scale} = \frac{\text{length scale}}{\text{virtual velocity}}
\]

‘Virtual velocity’ refers to the apparent (or average) rate of movement of material through the system, including the time spent in storage. Sediment particles, in fact, remain at rest during most of their journey through the landscape, only rarely undergoing actual transport. The timescale associated with the virtual velocity can be thought of as the ‘transit’ time of sediment through the system. The transit time is the characteristic (average) time that it takes sediment to move through the system (e.g., a hillslope or a drainage basin). This provides some benchmark for defining timescales for a study. Ranges of timescales that are appropriate for the study of geomorphological processes at each of the spatially defined study units of landscape evolution introduced in the preceding section are presented in Table 1.

An understanding of sediment residence times is necessary to estimate virtual velocities. Given the extreme rapidity of many significant transport processes (e.g., landsliding, debris flows and fluvial transport during significant flood events) when they actually occur, it is time spent in storage that largely determines the

<table>
<thead>
<tr>
<th>Study unit</th>
<th>Ranges of diameter for study unit (km)</th>
<th>Ranges of virtual velocity(^a) (km yr(^{-1}))</th>
<th>Timescale (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tectonic</td>
<td>Lower: (10^1)</td>
<td>(10^{-6} - 10^0)</td>
<td>(10^1 - 10^7)</td>
</tr>
<tr>
<td></td>
<td>Upper: (10^3)</td>
<td></td>
<td>(10^3 - 10^9)</td>
</tr>
<tr>
<td>Drainage basin</td>
<td>Lower: (10^0)</td>
<td>(10^{-6} - 10^0)</td>
<td>(10^9 - 10^6)</td>
</tr>
<tr>
<td></td>
<td>Upper: (10^3)</td>
<td></td>
<td>(10^3 - 10^9)</td>
</tr>
<tr>
<td>Hillslope</td>
<td>Lower: (10^{-1})</td>
<td>(10^{-6} - 10^{-2})</td>
<td>(10^1 - 10^5)</td>
</tr>
<tr>
<td></td>
<td>Upper: (10^1)</td>
<td></td>
<td>(10^3 - 10^7)</td>
</tr>
</tbody>
</table>

\(^a\)Virtual velocities cover a broad range owing to the range of possible processes associated with this parameter. Therefore, the timescales show a broad range. In order to ensure that significant landscape changes are observed, the upper ranges of timescales may be the most appropriate for landscape evolution studies.
rate at which material is evacuated from the system. Moreover, accumulations of stored sediment define many of the geomorphologically significant elements of the landscape. The dating of sedimentary surfaces and deposits, the results of which can be used to infer long-term storage times of sediment, has been a major component of historical studies in geomorphology. However, a reconciliation of the relative roles of movement and storage has not generally been forthcoming.

Residence times need to be evaluated for sediment subsisting on hillslopes, in valley flats, and in the active fluvial channel zone. Sediment may reside on hillslopes for long periods before being transported to the valley flat. If sediment then enters the active channel and remains a part of the active sediment load, it may be moved quickly through the fluvial system. However, if sediment entering the valley flat does not enter the active fluvial system, it may enter long-term storage along the footslope. Furthermore, sediment that enters the active system, but is then deposited during an aggradation phase, may also go into long-term storage in the floodplain. Significant lateral or vertical erosion by the river into the valley fill is required to entrain such material into the active channel system. The distribution of sediments amongst long- and short-term reservoirs significantly influences the patterns of landform evolution (Kelsey et al., 1987). If long-term reservoirs sequester a significant proportion of the sediment moving through the landscape, then rare, high magnitude events will have the dominant influence on landscape modification.

Studies of the distribution of sediment storage times are relatively uncommon. River floodplains have been most investigated (e.g., Everitt, 1968; Nakamura et al., 1995). It is found that the age distribution of floodplain deposits is approximately exponential (Figure 2) and, furthermore, the rate of floodplain erosion declines exponentially with increasing age (Nakamura et al., 1995; Nakamura and Kikuchi, 1996), presumably because the remaining stored material becomes steadily more remote from the locus of frequent disturbances. A consequence of exponential storage times is that the effects of large sedimentation events (which must be large erosion events elsewhere in the landscape) may persist for a long time. This is a manifestation of the so-called Hurst effect (see Kirkby, 1987a). An important implication of it is that, wherever significant sediment stores occur, no likely sequence of sediment transfer observations (generally such sequences are restricted to a few decades length, or less) is likely to encompass the full possible variability of the process, nor to have included the possibly most significant events in the long term.

Sediment budget studies provide a framework for the study of sediment storage (e.g., Dietrich and Dunne, 1978; Reid and Dunne, 1996). Further research in this direction, with a particular focus on the evaluation of long-term residence of stored material (e.g., Macaire et al., 2002), is required in order to improve understanding of sediment routing and its associated timescales.

III Numerical modelling of landscape evolution

1 The role of numerical modelling

There are two general purposes for constructing and testing a model realization of some process or system: (1) as a test of understanding of the process, as embodied in the statements incorporated into the model; and (2) to predict modelled system
behaviour. Models are, by their nature, imperfect representations of reality – reduced and, initially, conjectural representations of what we envisage as the real system. What, then, can numerical modelling contribute to the study of landscape evolution? Landscape models can be used to support or more thoroughly explore ideas and hypotheses that have been partly established in other ways (Oreskes et al., 1994). For example, transport relations, which may have been shown to provide reasonable results either in the field or experimentally in the laboratory, can be explored more fully in a model. Of particular importance is the possibility to examine the implications of long-continued operation of such processes.

A model can, of course, be analysed in any situation in which the governing process equations can be integrated analytically. But in landscape studies, only a limited number of two-dimensional special cases are likely ever to be analytic. Numerical modelling permits the exploration of joint or sequential action by several processes (geomorphological and/or tectonic) in a distributed field. This exercise usually confounds our analytical abilities and often our intuition. Hence, a modelling exercise provides an approach to assess the plausibility of ideas about generic or historical landscapes. Perhaps the most important role for models in this respect is as a tool for the exploration of various ‘what-if’ questions (Oreskes et al., 1994). Various controlling variables can be held constant, while others are allowed to vary. Sensitivity analyses can be performed by changing the nature or intensity of various processes and observing the effects on the morphological evolution of landscapes. Such an exercise is often possible only within a numerical modelling framework.

Finally, numerical landscape models can be used to explore theoretical ideas and conceptual models about which there is much conjecture, but little quantitative research. For example, Kooi and Beaumont (1996) explored the ideas of Davis and other early conceptual modellers using their numerical model.

**Figure 2** Exponential decay of remaining sediment volume with age measured in several river floodplains (data compiled by J.R. Desloges, University of Toronto). Fraser River data represent the upper Fraser River above Moose Lake. Grand River data are for locations below Caledonia, Ontario.
Any of these purposes implies a scale specification for landscape modelling because the appropriate characterization of the system is affected by scale and because the initial data requirements are thereby set. In no case can the objective be to replicate exactly the details of development of a particular landscape, since the boundary conditions and history of contingencies that have affected the landscape cannot practically be reconstructed.

2 Process specification

The geomorphological rules adopted in numerical models of landscape evolution rest on weak foundations. Many questions about both the appropriate physical representation and rates of transport processes over large scales, and even smaller scales, remain unanswered. Moreover, many geomorphological relations assumed in the literature to be true have not been subjected to rigorous evaluation. For example, a linear relation between gradient and soil creep has been posited and incorporated into landscape evolution models (e.g., Moretti and Turcotte, 1985; Koons, 1989; Anderson, 1994; Kooi and Beaumont, 1994), but has not been demonstrated at landscape scale. Indeed the scanty available evidence appears to contradict it (Kirkby, 1967; Martin and Church, 1997; Roering et al., 1999, 2001). Nor have the circumstances in which a linear model might provide a suitable approximation for modelling purposes been considered critically. This situation arises because of the difficulty to make observations of process over extensive areas, and to sustain measurements for an appropriate period.

Investigators of geomorphological processes have either adopted a full Newtonian mechanics framework for the definition of transport equations, or they have fallen back on scale correlations amongst certain system driving variates and ones describing summary system responses (consider, for example, sediment transport rating curves). But as the scale of study increases, it is not possible to resolve the same degree of mechanical detail as at small scales. Appropriate process specification requires that the level of detail incorporated into equations be adjusted for the particular scale of a study. In some cases, a strictly mechanistic approach may be appropriate while, in other cases, generalized approaches to process definition – even to the level of scale correlations – must be adopted.

At the hillslope scale, the frequency of grid points for the evaluation of elevation changes in numerical models can be relatively dense and, therefore, relatively small changes in morphology may be resolvable. The researcher may preserve a fairly detailed representation of the mechanics of processes operating on the hillslope and variables required for calculations. The parameterization may include strictly mechanistic terms such as shear and normal stresses, pore water pressures, and the like (see, for example, studies by Montgomery and Dietrich, 1994, and Wu and Siple, 1995). A mechanistic expression for the strength of surficial material on slopes is the Mohr–Coulomb equation

\[
s = c' + c_r + [(1 - m)\rho_b gd + m(\rho_{sat} - \rho)gd]\cos^2 \theta \tan \phi'
\]

in which \(c'\) is the effective material cohesion, \(c_r\) is pseudocohesion provided by root strength, \(\rho_b\) and \(\rho_{sat}\) are material bulk density and density of saturated material respectively, \(d\) is the depth of surface material above the potential failure plane, \(\theta\)
is the hillslope angle, \( m \) is fractional saturation (in effect, \( d_{\text{sat}}/d \), where \( d_{\text{sat}} \) is the depth of the saturated zone), \( g \) is gravity and \( \phi' \) is the effective angle of shear resistance of the material. This quantity is compared with the driving force for failure, the shear stress (\( \tau \)), on a candidate failure plane

\[
\tau = [(1 - m)\rho_b + m\rho_{\text{sat}}]gd\sin \theta \cos \theta
\]

(2)

\( F = s/\tau \) indicates the likelihood for failure, which becomes probable as \( F \) declines below 1.0. Neglecting cohesion

\[
F = \frac{[(1 - m)\rho_b + m(\rho_{\text{sat}} - \rho)]/[((1 - m)\rho_b + m\rho_{\text{sat}})](\tan \phi')}{\tan \theta}
\]

(3)

In a landscape development model, such an equation would be made operational either by keeping a spatially distributed water balance (entailing a hydrological submodel with considerable detail), or by using some stochastic assignment of \( d_{\text{sat}} \) at sites where \( \theta \) approaches or exceeds \( \phi' \). Separate rules are required in order to distribute the failed material downslope (in effect, to decide the character and mobility of the failure, whether slumps, debris slides or debris flows).

Minor, semi-continuous processes of soil displacement (creep, ravel, animal activity) might be modelled as a linear diffusive process

\[
\frac{\partial z}{\partial t} = -\kappa \frac{\partial^2 z}{\partial x^2}
\]

(4)

in which \( \kappa \) is the diffusion coefficient (a constant).

It is feasible to consider spatial variation of the phenomena described by eqs (1)–(4) because of a relatively high sampling frequency (of the topographic surface) in a hillslope-scale study. Nonetheless, it may still be difficult to retain all of the true complexity of the problem. Climate and vegetation may be treated as independent parameters that are approximately constant over relevant timescales, but hydrologic and vegetation characteristics may vary in space along the hillslope profile (see, for example, Ambroise and Viville, 1986). Vegetation, in particular, presents difficulties because there remains no general physical formulation of its effects even at the scale of geotechnical modelling of hillslope condition, because it is a dynamic factor, and because – over almost any scale of landscape interest – it responds to local climatic and hydrological variations.

As the scale of study is increased to that of drainage basins, local irregularities in morphology can no longer be computationally preserved, and are no longer of particular importance in the functioning of the system. Highly detailed process equations are no longer suitable as it is impossible to achieve correspondingly detailed knowledge of controlling variables because of difficulties associated with sparse sampling and error propagation over extended time and space scales. At geomorphologically significant timescales in sufficiently steep terrain, for example, minor soil disturbance might be ignored since the downslope displacement of surficial material over significant periods of time comes to be dominated by discrete failures such as debris slides (see, for example, Roberts and Church, 1986). These events, modelled discretely at hillslope scale, might now be simulated as a diffusive process under the assumption that, after a sufficient lapse of time, they will have affected
nearly every position on a slope. However, a significant threshold for slope instability remains, so the process cannot be considered to be linear. A more suitable formulation would make \( \kappa = \kappa(z, z_{xi}, t) \) yielding the nonlinear process

\[
\frac{\partial z}{\partial t} = \frac{\partial}{\partial x_i}[\kappa(z, z_{xi}, t) \partial z/\partial x_i]
\]

with specific, nonlinear functional dependences for \( \kappa \) (e.g., Martin and Church, 1997; Roering et al., 1999).

A similar exercise of generalization can be entertained for fluvial sediment transport. The downslope directed force of flowing water over the surface is given, as a local average, by

\[
\tau = \rho_f g d \sin \theta
\]

in which \( \rho_f \) is fluid density, \( d \) is water depth, \( \theta \) is channel gradient, and the quantities are specified for a unit width of channel. (The equation is simply a specification of eq. (2) for water flowing in a channel of modest gradient.) It has been found that, for the transport, \( g_b \), of bed material (material that may form the floor and sides of the channel)

\[
g_b = \mathcal{J}(\tau - \tau_0)^n
\]

in which \( \tau_0 \) is a function of material properties and of channel hydraulics, and \( n \geq 1.5 \) (commonly considerably greater in gravel-bed channels) is an exponent that varies with the intensity of the transport process. To use this formulation in a model, one requires a specification of flow depth, \( d \), and channel width and, to determine the threshold condition, probably a specification of the other principal hydraulic quantities, in particular flow velocity. One must also track material properties, particularly grain size. This requires explicit models of both hydrology and channel hydraulics. For most landscape modelling purposes, these requirements are beyond the limit of resolution.

In such cases, sediment transport is defined using generalized formulae. R.A. Bagnold’s approach (Bagnold, 1966, 1980) is commonly adopted to estimate total sediment transport as

\[
G_b = \mathcal{J}(\rho_f g Q \theta)
\]

in which \( Q \) is streamflow. The only other physical variate required is gradient, \( \theta \), which is a principal variate of any landscape model. So this represents an attractive alternative, provided a threshold for transport can also be specified in terms of \( Q \). At very large scales, this approach can expediently be reduced to an empirical correlation

\[
G_b = \mathcal{J}(Q, \theta)
\]

In some models for which the principal focus of attention is tectonic effects over long timescales, there is no explicit representation of alluvial processes and storage at all, which translates into an assumption that material, once it reaches slope base, is removed from consideration (e.g., Koons, 1989; Anderson, 1994). This strategy
amounts to an assertion that the transit time for sediment in the fluvial system is smaller than the model time step. It is not clear that this actually is true even for very large-scale models.

Glacial, aeolian and coastal processes have been comparatively little considered in landscape modelling studies, but it is certain that a similar scheme of appropriate generalization of the process representation will have to be developed in such cases (see Ashton et al., 2001, for an example that considers long-term, large-scale coastal development; see MacGregor et al., 2000 for an example considering longitudinal profile evolution of glacial valleys). Nearly all published landscape models adopt some variation on the scheme of equations presented above, with the addition of specific terms to cover effects such as tectonic forces and isostasy. An interesting example of such an additional, essentially independent process, is large-scale, rock-based failure (Hovius et al., 1997, 2000; Densmore et al., 1998). Practically, such events can be dealt with as shot noise under some suitable stochastic scheme (e.g., Dadson, 2000).

Over long timescales, it is additionally important to consider rock weathering. In a geomorphological model, this is the sole source of additional material that can be mobilized. This topic has been little studied, but a mathematical model based on field observations was introduced by Heimsath et al. (1997).

Major climatic, vegetation and geological changes should also be considered at large scales, although the large space and time steps in such models allow recognition of only relatively significant changes. Schumm and Lichty (1965) considered climate to be an independent variable at cyclic timescales (see Rinaldo et al., 1995, for a model study), but research has suggested that there may be very complex feedbacks and interactions between landscape evolution and climate change (Molnar and England, 1990). Furthermore, variables that are considered to be independent at the hillslope scale may have to be considered dependent variables at larger scales. For example, vegetation may be an independent variable for a study bounded within the slope. As scales increase and the possibility of spatially varying climate and long-term climate change is introduced, vegetation will vary in response to the climatic forcing.

3 Model implementation

A useful framework for implementing numerical models and assessing the limitations associated with them in the Earth sciences was presented by Oreskes et al. (1994) under the headings validation, verification, calibration and confirmation.

Validation strictly requires that a model contain no known flaws, be internally consistent, and produce results that are consistent with known prototypical instances. A flawless model would be a definitive representation of the physical world as we currently understand it (but even that is not a guarantee that it faithfully represents the real system). Models in Earth science are never definitive because of the complexity of the systems being studied. Indeed, the view that we are advancing here is that the art of Earth system modelling is to judge what ‘flaws’ must be tolerated within the model, for the sake of computational tractability, without compromising the essential phenomena that are the object of the modelling exercise. Pragmatically, the necessity for this judgement arises in any scientific exercise. Whereas the object in analysis is to
come as close as possible to a definitive representation of the phenomena, in the
synthesis of complex systems the constraints of scale and information compel this
judgement to be the central step of the exercise.

In practice, then, validation in Earth system models is about: (i) maintaining
internal consistency in the model representation and (ii) ensuring that the model
laws represent some suitably ‘average’ behaviour of the truly more complex pro-
cesses. Practically, the former amounts to ensuring that continuity is preserved in
flux relations, which usually comes down to ensuring computational closure. This
is a purely technical matter that we will not pursue. The latter remains a challenge,
due in large part to the paucity of field data available about a particular process over
the range of scales that would be necessary to test the requirement rigorously. In
practice, ‘all known prototypical instances’ usually amount to an individual
datum or data set, and data for different parts of a model are often assembled
from different landscapes.

Model verification refers to the process of determining the ‘truth’ of the model.
This is a stronger condition than validation and it cannot in principle be achieved
except in closed systems of deduction. More prosaically, Earth system models are
generally considered to be ‘open’ because of incomplete knowledge of input para-
eters. The models are underspecified. This occurs because of (i) spatial averaging
of input parameters; (ii) nonadditive properties of input parameters; and (iii) the
inferences and assumptions underlying model construction. This ‘incompleteness
of information’ means that model verification is not possible in open systems.

An incomplete model requires calibration. Model calibration involves the manipu-
lation of adjustable model parameters to improve the degree of correspondence
between simulated and reference results. However, arrival at consistent results
does not imply that a model has been successfully validated or verified. Calibration
of a model is often undertaken empirically, without a theoretical basis for the adjust-
ment. A model that has been calibrated to a certain set of data may not perform
adequately for other data sets.

In landscape modelling there are, in general, two strategies available for cali-
bration. The first entails initializing the model at some arbitrarily defined surface
configuration, and running it with set parameters to some reference state (for
example, the actual contemporary configuration of the reference surface). This pro-
cedure is analogous to the way in which other Earth system models (notably climate
models) are calibrated. The reference state is assumed to be an equilibrium state
of the system. In most cases, this would be an extremely difficult exercise because
of the intimate interplay between immanent and contingent processes in determin-
ing the state of any particular landscape. To our knowledge, this has rarely been
attempted. An example that discusses the problem of model initialisation
in some detail is given by Ferguson et al. (2001). These authors simulated the rela-
tively short-term development of an aggrading river reach in what amounts to a
‘hillslope-scale’ model and their principal effort was directed at model convergence
onto the current state of the prototype surface, beginning from an arbitrary initial
state. The drainage initiation problem is a major modelling problem in geomorpho-
logy that is usually set up to follow the formal procedure discussed here, but it is not
run to match a specific landscape.

The second available procedure is to set process constants within the model to
values derived from field measurements of processes. Remarkably, apart from the
specification of certain tectonic rates in full models, published models have not been able to constrain geomorphological process rates with any reasonable degree of certainty. For example, many models have adopted diffusion to simulate various hillslope processes, such as landsliding or soil creep. However, values for the diffusion coefficients implemented in models have often been based on scarp studies, which are probably not representative of the processes in operation in the larger region being modelled. In addition, the climatic or geologic controls are likely to be very different. Some more recent studies have attempted to calibrate hillslope transport equations adopted in landscape models. For example, Martin (2000) used an extensive landsliding data base (Martin et al., 2002), based on aerial photographic analysis and accompanying field data, to calibrate transport equations in a study of hillslope evolution for coastal British Columbia. Nonetheless, there remains a lack of knowledge regarding transport rates for hillslope processes operating over medium to long timescales.

Similarly, constants found in stream power relations used to simulate bedrock or alluvial processes are often not based on strong evidence. Indeed in many cases the choice of constants is not discussed and hence the equations have not necessarily been calibrated effectively. Snyder et al. (2003) attempted to overcome this shortcoming by undertaking an empirical calibration of the stream power equation for bedrock incision using field data from northern California.

An ostensible reason for the shortcoming of adequate calibration in landscape modelling is the dramatic disparity between the record of almost any contemporary measured rates and the timescale of landscape models. There can be no assurance that contemporary processes conform with averages that may hold over landscape--forming timescales. Church (1980) and Kirkby (1987a) argued that relatively short-term records of significant landscape forming processes are very unlikely to demonstrate the full range of variability that is ultimately experienced (see also Kirchner et al., 2001, for an empirical example). Moreover, the pervasiveness of human disturbance over much of the terrestrial surface today is apt to yield a suite of highly nonrepresentative values for many landforming processes. Nonetheless, careful use of available data remains the best approach to model calibration that is available.

4 Testing outcomes

Whereas model verification and validation, according to the strict definitions given above, are not possible, model confirmation remains an achievable objective. Confirmation is established by examining the ability of a model to match prediction (model outcome) with observation in some formally defined manner.

There has been almost no methodical study of this issue and no consensus as to what constitutes a sufficient test of a model’s performance. A number of approaches towards model testing have been adopted. On one end of the spectrum, some models have been used primarily as an exploratory tool to examine the behaviour of various processes and no strict attempt is made to evaluate the resemblance of model landscapes to real landscapes. For example, Tucker and Bras (1998) used their model of drainage development primarily as an exploratory tool to examine the operation of several hillslope process laws. In other modelling studies, a specific landscape or type of land-
scape is simulated and qualitative comparisons are made between simulated and real landscapes. In such cases, qualitative consistency with the modern landscape may be achieved (e.g., Howard, 1997). The study by Ferguson *et al.* (2001) is a rare example of an attempt at full quantitative comparison, but it confines itself to a very local problem. Finally, some studies have compared statistical properties, such as hypsometry, between simulated and real landscapes (e.g., Anderson, 1994). Willgoose and his colleagues have made significant advances in this area of research, using landscape statistics and experimental simulators to test model landscapes (e.g., Willgoose, 1994; Willgoose and Hancock, 1998; Hancock and Willgoose, 2001).

**IV Approaches to modelling landscape evolution**

This section reviews key developments in modelling the geomorphological component of landscape evolution for three categories of models (1) conceptual (2) quasi-mechanistic and (3) generalized physics. The list of landscape evolution models (Beaumont *et al.*, 2000) is now so extensive as to preclude detailed discussion of all individual models.

1 **Conceptual models**

Conceptual models of landscape evolution were created between the 1890s and the mid-twentieth century (Davis, 1899; Penck, 1953; King, 1962). The point was to ‘explain’ the observable end products of long-term landscape evolution and, hence, modelling often relied on space–time substitution (Paine, 1985; Thorn, 1988). W.M. Davis (1899) proposed that, after a period of brief and episodic uplift, landscapes underwent downwearing through a series of predictable stages referred to as the ‘cycle of erosion’. Numerous summaries and critiques of Davis’ work are available in the literature (e.g., Chorley, 1965; Flemal, 1971; Higgins, 1975). In contrast, Penck (1953) rejected the notion of disjunct uplift and erosional events and instead focused on their continuous interaction. He advocated the concept of slope replacement whereby the steep part of a slope retreats rapidly and leaves behind a lower angle debris pile at its base.

A frequent criticism of these early models is that exogene processes were treated in a superficial, nonquantitative manner and that endogene processes were poorly represented. But these criticisms are made in light of much increased understanding of landscape forming processes. The significance of these conceptual models is that they provided an initial set of ideas of how landscapes might evolve at large scales. The processes described in the models nearly always amounted to conjecture on the part of the modellers, and the observations on which the models were based were usually strictly morphological.

2 **Quasi-mechanistic models**

After the mid-twentieth century, an entire generation of Anglo-American geomorphologists focused almost exclusively on investigations of erosion, transport and deposition of sediments within a mechanistic framework over relatively small scales (see Church, 1996). Consequently, a wealth of process knowledge became available for incorporation into quantitative models of geomorphological change.
Accordingly, the landscape models of Kirkby (1971) and Ahnert (1976) incorporate quasi-mechanistic process-equations. The term quasi-mechanistic is used here to signal the fact that, although the equations do rely on a significant degree of empiricism, an attempt is made to express process operation for individual processes in a mechanistic manner, often including as much detail as knowledge at the time allowed. In this kind of reductionist approach, overall system operation is resolved by summing individual behaviours of lower-level subsystems.

In the models of Ahnert (1976) and Kirkby (1971, 1987b), a series of geomorphological process equations, including such details as slope wash and rainsplash, are applied to the landscape surface, but calibration of equations and definition of key coefficients and exponents are not discussed. Ahnert developed a computer program to run his model, increasing the efficiency of calculations, whereas Kirkby, in his early modelling efforts, kept the mathematics tractable (and sometimes analytic) by focusing on the evolution of hillslope profiles. The models were used to assess landscape changes that would result when one process was operating or when several processes were operating in combination. In this manner, various ‘what-if’ questions were addressed. Both researchers continued to develop their models in subsequent years (e.g., Ahnert, 1987; Kirkby, 1992).

Despite the obvious attraction of quantitative rigor, the quasi-mechanistic approach to process constrained the scale, suggesting that the models may not be suitable for long timescales. Ahnert nevertheless has applied his model at a variety of scales ranging from individual hillslopes through to entire mountain ranges (Ahnert, 1987) even though it seems unreasonable to apply this one representation of the physics to such a large range of scales.

3 Generalized physics models

a Overview: Generalized physics models involve deliberate attempts to simplify the representation of what are known to be a large number of individually complex processes to a level of detail appropriate for extended scales of space and time. Early models of this type focused on the development of hillslope profiles and so were strictly geomorphological models. For example, Culling (1960, 1963, 1965), Luke (1972) and Hirano (1975) recognized the potential of diffusion-type relations to model transport processes over large scales. The analytical solution of equations, necessary at the time, made them cumbersome to manipulate and restricted model complexity.

The evolution of landscape surfaces can now be examined using numerical techniques to solve the relevant equations very efficiently within a computer. Many of the recent models are used to assess the development of landscapes strongly influenced by tectonics, such as foreland basins, rift escarpments and collisional or convergent mountain belts (e.g., Flemings and Jordan, 1989; Kooi and Beaumont, 1994; Willett et al., 2001). But models with a particular focus on geomorphological processes have also been developed (e.g., Benda and Dunne, 1997; Tucker and Bras, 1998).

Both hillslope and fluvial processes are incorporated in the surface processes component of most of these recent models, with diffusion-type equations and stream power relations typically being used to simulate them. If grid cells are of a large size, for example >1 km², fluvial and hillslope relations are often applied across
each cell. However, the development of models utilizing triangulated irregular networks (TIN) grids has allowed for greater flexibility in defining discrete hillslope and channel cells within a model (e.g., Tucker et al., 2001). Research has taken place in recent years that has begun to provide the necessary underpinning for calibrating geomorphological process equations within landscape models (e.g., Seidl et al., 1994; Heimsath et al., 1997; Hovius et al., 1997, 2000; Martin, 2000; Snyder et al., 2003). Defining effective ways to test models remains an outstanding issue (see Section III, 4).

b Key developments: The prominence of diffusion-type equations is evident in several of the key models that emerged in the late 1980s (e.g., Koons, 1989; Anderson and Humphrey, 1989; Flemings and Jordan, 1989). Anderson and Humphrey applied their model to a range of scales, from small-scale hillslope development to larger-scale basin sedimentation, whereas Flemings and Jordan focused on sediment delivery to foreland basins. Koons examined the evolution of the Southern Alps of New Zealand, a collisional mountain belt. Uplift was included in the models of Koons, and Flemings and Jordan.

Several landscape evolution models (Anderson, 1994; Kooi and Beaumont, 1994; Tucker and Slingerland, 1994) were published in a special issue of the *Journal of Geophysical Research* (1994). These models emphasized interactions between endogene and geomorphological processes, and the development of particular tectonic features over timescales of the order of 1 Ma was explored. Kooi and Beaumont (1994) and Tucker and Slingerland (1994) explored the evolution of escarpments related to rifting, whereas Anderson (1994) explored the evolution of the Santa Cruz Mountains. In the model of Kooi and Beaumont, the cumulative effect of hillslope processes (including slow, quasi-continuous and rapid, episodic mass movements) is simulated using linear diffusion. Anderson (1994) and Tucker and Slingerland (1994) used linear diffusion to simulate slow, mass movements with additional algorithms defined to model large hillslope failures. Tucker and Slingerland (1994) incorporated a weathering rule in their model to monitor the supply of sediment. Variants of stream power relations are used in all of these models to simulate fluvial transport processes for alluvial and/or bedrock channels. Models focusing on interactions between tectonics and surface processes continue to emerge up to the present day, each adopting slightly different, generalized approaches for the geomorphological component. For example, Avouac and Burov (1996) emphasized the role of linear and nonlinear diffusion while Willett (1999) focused on bedrock channel incision in the evolution of intracontinental and convergent mountain belts. Other researchers continued to expand and refine the specification of key geomorphological processes in particular tectonic settings. For example, Densmore et al. (1998) included a detailed rule for deep-seated bedrock landsliding, which had been lacking in earlier models, in their model of the normal-fault-bounded mountains in the Basin and Range over timescales up to 1 Ma.

Models have also been developed and refined to examine classic, long-standing concepts in the landscape literature. For example, Kooi and Beaumont (1996) used their landscape development model to explore the conceptual models of Davis, Penck and King. A significant number of studies have considered the concept of
steady state in mountain evolution (Whipple and Tucker, 1999; Montgomery, 2001; Whipple, 2001; Willett et al., 2001). Willett and Brandon (2002) recognize that, while steady state is often invoked in combined tectonic/geomorphological models, the particular meaning attached to the term is not always clear. They use their landscape model to provide clarification of possible ways to invoke steady state when considering the formation of mountain belts. This is an interesting enquiry because it confronts Hack’s (1960) assertion about equilibrium in landscape evolution with a critical test which, if confirmed, could become the basis for a general strategy to obtain model confirmation (cf. Section III, 4).

In recent years, several models have been published that have highlighted the geomorphological component of landscape evolution, with endogene processes, if included, being secondary in importance (e.g., Howard, 1997; Benda and Dunne, 1997; Tucker and Bras, 2000). Typical timescales for application of these models range from $10^3$ to $10^4$ years. Benda and Dunne (1997) developed a model to examine the development of the Oregon Coast Range over about an 8000-year time period, focusing on stochastic, climate-driven forcing of sediment supply to channel networks. Howard (1997) considered the development of the Utah badlands over timescales up to many tens of thousands of years, employing diffusion and threshold-driven equations to simulate slow mass movements and rapid failures respectively, and stream power equations for transport in bedrock and alluvial rivers. A stochastic rainfall component was incorporated to drive geomorphological processes in Tucker and Bras (2000).

The implementation of irregular grids in numerical models of landscape change represents a significant development (e.g., Braun and Sambridge, 1997; Tucker et al., 2001). This approach allows variation of grid resolution across the landscape surface; for example, a higher grid resolution is usually preferred along channels than on interfluves. Braun and Sambridge (1997) illustrated the improved model performance obtained when adopting irregular spatial discretization of the grid. Tucker et al. (2001) implemented triangulated, irregular networks in their hydrologic/geomorphological model. It appears that this will be a feasible means to overcome the problem of different spatial resolution required to model ‘slow’ or episodic processes on hillslopes, in comparison with the ‘fast’ processes in river channels, at the same scale of temporal resolution. This has been one of the most significant practical problems to hinder landscape modelling efforts with geomorphological resolution.

V Conclusions

The modelling of landscape evolution has been made quantitatively feasible by the advent of high speed computers that permit the effects of multiple processes to be integrated together over complex topographic surfaces and extended periods of time. The development of this capability coincided with a period of intense focus, in the community of geomorphologists, on process models for sediment movement. A few pioneers (notably F. Ahnert and M. Kirkby) took up the challenge to create geomorphological models of landscape evolution. Geophysical interest in surface processes as a significant aspect of the long-term, very large-scale evolution of Earth’s surface, has helped to renew geomorphological interest in landscape devel-
opment. Today a range of diffusion-advection models exists for the study of fluvial landscapes and models for other geomorphological processes are beginning to appear.

The models emphasize, however, the dramatic shortage of data about geomorphological processes over timescales sufficient to be used for sensitive calibration or critical testing of any model. The message here is that renewed field activity will be important in the years ahead; field activity directed by the identification of critical data requirements for modelling. This activity will require a reintegration of process studies and historical investigations in the subject, since it is clear that data sets of sufficient length to inform even models with quite high temporal resolution are almost completely nonexistent. The ball remains in the geomorphologists’ court. Playing the ball is apt to lead to a reform of the subject that goes far beyond the adoption of a capability to construct numerical models of landscape evolution.

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