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Spatio-temporal scaling of channels in braided streams

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Abstract

The spatio-temporal scaling relationship for individual channels in braided streams is shown to be identical to the spatio-temporal scaling associated with constant Froude number, e.g. $F_r = 1$. A means to derive this relationship is developed from a new theory of sediment transport. The mechanism by which the $F_r = 1$ condition apparently governs the scaling seems to derive from the sensitivity of sediment transport to flow fluctuations when $F_r = 1$. The condition $F_r = 1$ is also given a theoretical basis using arguments from surface roughness.

Keywords: Frequency; Magnitude; Earthquake

1. Introduction

Scaling relationships are frequently observed in geophysics, such as frequency–magnitude relationships of earthquakes (Turcotte, 1997, 2004), the power spectrum of turbulence (Kolmogorov, 1941), or peak flow frequencies in rivers (Gupta, 2004). Such relationships may be derived from, e.g. fractal structures, cellular automata, or dimensional analysis (Kolmogorov, 1941). When dimensional analysis suggests a scaling relationship, additional theoretical developments are usually sought.

Experimental work by Foufoula-Georgiou and Sapozhnikov (2001) on the dynamics of braided streams and their constantly shifting channels have shown a very consistent spatio-temporal scaling result, that the length of a channel, ζ, is proportional to the square of its lifetime, τ^2, or

$$\frac{\varsigma}{\tau^2} = \text{constant} \tag{1}$$

The lifetime of a channel is the length of time that it is active. Observations indicate that dynamics of smaller channels is regulated by boundary conditions imposed by larger channels (Foufoula-Georgiou and Sapozhnikov, 2001). This may be interpreted as complexity, i.e. that the system dynamics be constrained importantly by the boundary conditions.

Expressed as dimensional analysis, rather than as variables with specific interpretation, the scaling result of Foufoula-Georgiou and Sapozhnikov (2001),

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Eq. (1), looks like

$$\frac{\Xi}{T^2} = \text{constant} \tag{2}$$

with Ξ length and T time. We will argue that for braided streams the Froude number, F_r, is a constant,

$$F_r^2 = \frac{u*^2}{2gh} = \text{constant} \tag{3}$$

In Eq. (3) $u*$ is the friction velocity, g is the acceleration due to gravity, and h is the depth of stream. Dimensional analysis of Eq. (3) also yields Eq. (2); since g is a *constant* acceleration, the combination of units in $u*^2/h$ must be also. Constant acceleration means the constancy of a ratio of length to time squared, Eq. (2). The equivalence of the two results of dimensional analysis suggests that transport dominated by constant Froude number may play a role in the dynamics of stream braiding.

The suspicion is strengthened by observations in steep, sand-bed channels, as found on high-energy Oregon beaches and in lahar runout channels draining Mt. Pinatubo following the 1991 eruption (Grant, 1997). In these channels, the Froude number of active channels and braids was generally very close to 1 (Fig. 1). The standing patterns evident in the figure occur in steep, coarse grained channels and are diagnostic of $F_r = 1$ (Tinkler, 1997a,b). When F_r dropped below 1, rapid channel silting led to channel abandonment and establishment of alternate channels. In this context we mention that there is extensive literature relating to optimization principles and $F_r = 1$ (Schoklitz, 1937; Inglis, 1947; Lamb, 1945; Jaeger, 1956; Chow, 1959; Chang, 1979; Rodriguez-Iturbe and Rinaldo, 1997; Huang and Nanson, 2000; Huang et al., 2003). The principle of maximum sediment transporting capability (Kirkby, 1977) would, for supercritical flow, lead to $F_r = 1$. In fact recent work has extended the optimization framework to determine that $F_r = 1$ also governs channels with friction and sediment transport (Huang and Nanson, 2000; Huang et al., 2003), though, for a theoretical treatment such as proposed here it is a drawback that these authors treated sediment transport empirically.

In the following, we look for a way to use a new theoretical approach to sediment transport in turbulent flow to find relationships between various length scales in fluvial sediment transport that account for an equivalence between Eqs. (1) and (3) that extends beyond mere dimensional analysis.

Fig. 1. Braided lahar runout channel, Pasig-Potrero River, Philippines. Note standing wave patterns in each of the braids, indicating near-critical flow conditions (Tinkler, 1997a,b).

2. Theoretical background

2.1. Probabilistic transport formulation

We consider an energy-based probabilistic approach to sediment transport introduced in Hunt (1999). The basis for the approach is a conceptualization of a stream as a collection of "bursts" of energy generated by flow turbulence. The initial ansatz was that such kinetic energy bursts follow Boltzmann statistics. The transfer of the kinetic energy to particles was assumed to occur suddenly, like the transfer of energy from the electromagnetic field (as photons) to electrons. If sufficient, this energy could then lift the particle over a potential barrier, entraining it in the turbulent flow. The argument of the exponential in the Boltzmann statistics was written as a ratio of a gravitational potential energy and a turbulent flow kinetic energy, analogous to the Boltzmann factor defining the distribution of molecules with height above the earth's surface (Reif, 1965).

The relevant potential energy for a particle of density ρ and volume V is $(\rho-\rho_w)Vgy$, where ρ_w is the density of water and y is the height lifted from its stationary position on the bed. The bed stress,

$$\Gamma = \frac{1}{2}\rho_w u*^2 \quad (4)$$

also has units of kinetic energy density and, multiplied by V, gives the expected energy in a volume V at the level of the river bed. The analog to the Boltzmann factor was constructed as,

$$P = \exp\left[\frac{-V(\rho-\rho_w)gy}{\frac{1}{2}\rho_w u*^2\, V}\right] \quad (5)$$

Subsequent development in (Hunt) 1999 demonstrates that the argument of Eq. (5) is correct, and provides evidence that the functional form is reasonable. For example, assume that all particles on the bed have the same diameter, d. In order for a particle to be entrained into the flow, $y \propto d$ but P drops from near 1 to near 0 at a radius d proportional to the square of the friction velocity, $u*^2$; this is in complete accord with the Shields' diagram.

Note that the general results for entrainment given below do not depend on the functional form of Eq. (5), only that P be a rapidly varying function of the given argument. On the other hand, the specific results for the roughness do depend on the functional form of Eq. (5), and may be in accord with empirical results summarized by Topping (1997).

2.2. Rate equation and rates

In Hunt (1999), P is used to set up a one-dimensional rate equation to determine the equilibrium particle size distribution on a river bed,

$$-PBN_b(d) + A[N(d) - N_b(d)] = 0 \quad (6)$$

in which turbulent settling and entrainment rates are equal for each particle size.

In Eq. (6) PB is the entrainment rate and A the settling rate, (both in probability per unit time) $N(d)$ the number of particles of diameter d present, and $N_b(d)$ the fraction of those on the bed of the stream. B was chosen as,

$$B = \frac{u*}{d} \quad (7)$$

the inverse of the lifetime of an eddy of the appropriate size. The inverse of B, $d/u*$, is also known as the bursting period, consistent with the interpretation that the energy is provided through kinetic energy bursts or fluctuations. The settling rate, A, was obtained by setting gravitational and turbulent forces equal,

$$d^2\rho_w v_y \frac{u*}{y} d = (\rho-\rho_w)d^3 g \quad (8)$$

with v_y the vertical velocity of the particle. A particle lifted into the stream a height y_m, can then settle out according to Eq. (8). The time of settling, t, is proportional to an integral of the inverse of the vertical velocity from the maximum height of entrainment, y_m, to the bed height, y_0,

$$A^{-1} \propto t = \int_{y_0}^{y_m} \frac{dy}{v_y} \quad (9)$$

The result of the integration, using Eq. (8) for the vertical velocity, is,

$$A \propto t^{-1} = \frac{(\rho-1)g}{u^*}\frac{1}{\ln\left(\frac{y_m}{y_0}\right)} \quad (10)$$

2.3. Sediment flux

Eqs (9) and (10) were utilized to find a one-dimensional equation for the effective downstream transport velocity of a particle of diameter d, provided d is small enough to be entrained (Hunt and Papanicolaou, 2003). To use Eq. (10) an explicit expression for y_m must be obtained.

Suppose a particle receives more kinetic energy from the stream than the minimum required for entrainment. Then y_m can be found by setting the excess kinetic energy equal to the potential energy at the maximum height of entrainment (Hunt and Papanicolaou, 2003),

$$y_m = \frac{1}{\rho}\left[\frac{(u^*)^2}{2g} - (\rho-1)d\right] = \frac{1}{\rho}[F_r^2 h - (\rho-1)d] \tag{11}$$

After substituting Eq. (11) into Eq. (10) for the distance per entrainment, we multiply by u^*/d, the attempt frequency, to obtain a quantity proportional to the total distance traveled (Hunt and Papanicolaou, 2003),

$$L \propto \frac{u*^3}{gd}\ln\left[\frac{u*^2}{gy_0} - \frac{d}{y_0}\right] \tag{12}$$

Since the units of the right hand side of Eq. (12) are velocity, Eq. (12) gives an effective transport velocity. Subsequent comparison with both experimental and field data for travel distance (Hunt and Papanicolaou, 2003), showed that the power of u^* in the correct result must be reduced by 1, i.e. meaning that the unknown proportionality factor in Eq. (12) was proportional to the inverse of u^*. If $y_0 \propto d_{50}$ (the median particle size on the bed) one can then write,

$$L \propto \frac{u*^2}{gd}\ln\left[\frac{a'u*^2}{gd_{50}} - \frac{d}{bd_{50}}\right]; \quad L = \frac{C\Gamma}{d}\ln\left[\frac{a\Gamma}{d_{50}} - \frac{d}{bd_{50}}\right] \tag{13}$$

In Eq. (13) a, a', b, and C were numerical parameters (Hunt and Papanicolaou, 2003). While the values of a, b, and C in the second formulation of Eq. (13) were not predicted, they were consistent across different scales and from field to lab experiments (Hunt and Papanicolaou, 2003). Eq. (13) was shown (Hunt and Papanicolaou, 2003) to give results in agreement with both field observations (De Vries, 2000) and experiment (Knapp, 2002).

2.4. $F_r = 1$ and surface roughness

The theoretical reason for choosing F_r as approximately 1 (Hunt, 1999) was as follows Using P, an expression was derived (Hunt, 1999) for the roughness of a stream bed with a single particle size. The root mean square fluctuation in bed elevation for high entrainment probability, consistent with $F_r > 1$, was found to be,

$$\frac{d}{\ln^{1/3}\left[\frac{\rho u*^2}{gd(\rho-\rho_w)}\right]} \tag{14}$$

The situation for low entrainment probabilities, $F_r < 1$, is discussed below. Expression (14) was then identified with y_0 and substituted into the vertical velocity distribution:

$$v_s = u* \ \ln\left[\frac{h}{y_0}\right] \tag{15}$$

for the velocity, v_s, at the water surface $y = h$ (Hunt, 1999). Consistent with the present 1D treatment, the product of v_s and h was set equal to the flow per unit width, q. The resulting expression for $v(h)$ as a function of q was similar to that obtained from minimization of the specific energy. Thus it was suggested (Hunt, 1999) that the configuration to which the bed evolves should be governed by the principle that the stream transfers as much energy to the bed particles as possible (for conditions that would otherwise produce supercritical flow). Such maximum roughness should also be consistent with maximum reduction of F_r towards 1.

3. Critical and supercritical flow with a mobile bed: limits on sediment transport

If F_r is nearly 1, Eq. (13) can be rewritten (Hunt and Papanicolaou, 2003),

$$L = \frac{Ch}{d}\ln\left[\frac{ah}{d_{50}} - \frac{d}{bd_{50}}\right] \tag{16}$$

In this form, Eq. (16) can be used to show that: (a) there is a maximum value of d that can be transported;

(b) there is a minimum value of d, for which the transport can be considered part of the bedload; and (c) both of these values are proportional to the depth of the stream, h. The logarithmic factor vanishes at

$$\frac{ah}{d_{50}} - \frac{bd}{d_{50}} = 1 \tag{17}$$

and the maximum d entrained is equal to the difference between a numerical factor times h and d_{50}. The minimum d that is part of the bed load is found by setting the distance of travel in a time t equal to the product of u^* and t. For larger values of L (smaller d), the particle would have to move downstream faster than the water just above the bed, meaning that this formulation for bedload transport must be replaced by a result for suspended load.

On a plot of L vs. log d, Eq. (16) shows up as sigmoidal in shape with steep (negative) slopes at both small and large d. Qualitatively, particles with large d remain stationary, while those with small d are transported out of the system. Thus in a given reach, far from the source of the largest particles, the range of particle sizes can be expected to be limited, with the limits determined in terms of h. Thus, if flow conditions persist, d_{50} must also be proportional to h, and the consistency of the parameters in the field measurements is not surprising.

4. Flow fluctuations and channel abandonment

Consider now a qualitative analysis of the stability of flow and bed roughness against fluctuations for $F_r > 1$. A fluctuation in Froude number associated with a local reduction in u^* would lead to slightly (on account of the logarithmic dependence) larger values of the equilibrium roughness. A larger roughness will lead to slower flow and greater sedimentation to entrainment ratio. This combination represents a (very weak) positive feedback. The conclusion is also generally consistent with the above deduction that the surface roughness contributes to a tendency to reduce F_r to 1, although it does not explain why $F_r < 1$ should not be expected. This result can only be understood if we look at the dependence of surface roughness on Froude number for $F_r < 1$.

Although no explicit result for the surface roughness for small entrainment probabilities was obtained in Hunt, 1999, Eq. (13) from that work can be used to generate such an expression. The result is,

$$y_0 \propto d\left[\frac{d(\rho-\rho_w)}{hF_r^2\rho_w}\exp\left(\frac{d[\rho-\rho_w]}{hF_r^2\rho_w}\right)\right]^{1/3} \tag{18}$$

Note that for distributions of particle sizes we will substitute the median particle size, d_{50}, for d. Compare Eq. (18) with Eqs. (3.58) and (3.59), empirical results quoted by Topping (1997), and summarized as

$$y_0 = z_0 \exp\left[\frac{5\rho}{\rho_w}(\langle\varepsilon_s\rangle)\right] \tag{19}$$

where $\langle\varepsilon_s\rangle$ is the time-averaged concentration of near-bed suspended sediment and z_0 is $d_{50}/30$, if there is no bedload saltation layer, but is roughly 1/20 of the thickness of that layer, if it exists. While the fundamental structure of Eq. (18) is thus compatible with empirical formulations, it is not clear whether Eq. (18) can actually serve as a predictor of field observations. Eq. (18) for y_0, a product of an exponential and a power is, in the case of low entrainment probabilities ($F_r < 1$), clearly a much more rapidly decreasing function of F_r.

Let's consider the possibility that a stream channel may become highly sensitive to small fluctuations in flow as the Froude number drops to near 1. This will require a somewhat more quantitative discussion. In a steep channel with high sediment supply, flow tends to be reduced to critical by developing bed roughness, including bedforms (Grant, 1997). Here we consider the mechanism for reducing F_r to near one by surface roughness alone; inclusion of bedforms in this formulation requires a much more complex treatment.

First consider the case of $F_r > 1$. A range of particle sizes limited at high and low d by different proportionalities to the channel depth is transported along the bed, larger particles are either not present, or not transported, and smaller particles are part of the suspended load. Although Eq. (14) represents the roughness for a single particle size, let's take it as a lowest order approximation for the roughness of a distribution of particle sizes as well, but now with d_{50} substituted for d. Consider just the largest particle suspended. From Eq. (13) one can write (analogously to Eq. (17),

$$d_{\max} = \frac{\rho_w u*^2}{2(\rho-\rho_w)g} - y_0 = \frac{\rho_w u*^2}{2(\rho-\rho_w)} - d_{50}\ln^{-1/3}\left(\frac{\rho_w u*^2}{2g(\rho-\rho_w)}\right) \tag{20}$$

Although for $F_r > 1$, a reduction in $u*$ generates a positive feedback, the one-third power of a logarithm is a very slowly varying function in $u*$, and it should be reasonable to approximate y_0 as a constant for most applications.

But if $F_r \leq 1$, what does a reduction in $u*$ produce? Now the surface roughness is a much more rapidly varying function of $u*$ and we find,

$$d_{\max} = \frac{\rho_w u*^2}{2(\rho-\rho_w)g} - y_0 = \frac{\rho_w u*^2}{2(\rho-\rho_w)g} - d_{50}\left(\frac{2g(\rho-\rho_w)}{\rho_w u*^2}\right)^{1/3}\exp\left[\frac{2g(\rho-\rho_w)}{3\rho_w u*^2}\right] \tag{21}$$

The product of an exponential and a power can no longer be approximated as a constant for any application. Now the roughness term is a more rapidly varying function of $u*$ than is the bed stress term, $u*^2$. Therefore, a small drop in $u*$ may produce a rapid change in both surface roughness and the largest sized particle that can be entrained. Of course it takes a finite time for the roughness to respond to the change in $u*$. But we expect that this can be accomplished by rapid sedimentation through the relatively high sensitivity of the bedload to $u*$ on account of the logarithmic cut-off in travel distance (Eq. (13)). Then the channel may be expected to rapidly silt up and fill in, necessitating the opening of a new channel. Thus there is a tendency for Froude numbers of channels to be reduced slowly through feedbacks to near $F_r = 1$, but then for the feedback to accelerate rapidly when the Froude number drops below 1, so that channels with $F_r < 1$ are quickly abandoned and effectively isolated from the system. This combination of factors should tend to accentuate the occurrence of channels with F_r near 1, as well as to rapidly remove channels with $F_r < 1$ from the system. According to empirical observations (i.e. Ergenzinger, 1987; Grant, 1997), this is exactly what occurs. Vincent and Smith (2001) inferred through both modeling and field work that the longitudinal profile of alluvial channels alternated from supercritical to subcritical flow with mean values of F_r over a reach equal to 1. The accompanying morphology alternated from wide and shallow to narrow and deep. The wide shallow portion changed from low gradient to high gradient, with the largest gradients just upstream from the subcritical flow regions. Channel branching occurred only in the upper portions of the supercritical flow regions, with smaller channel gradients and, it would appear, the values of F_r nearest 1. These authors asserted that $F_r = 1$ could be used to forecast flood threats in the southwestern USA.

Physically, then, one develops the picture that it is difficult to maintain supercritical flow for steep channels with mobile beds. The tendency for $F_r = 1$ makes channels especially sensitive to small fluctuations in flow, which could be brought about by a random coincidence of sedimentation of several larger particles, analogous to grain flow through a (narrow) hopper. The sensitivity to flow fluctuations at a critical value of the Froude number may allow an analysis based on self-organized criticality concepts (Sapozhnikov and Foufoula-Georgiou, 1996, 1999).

5. The Spatio-temporal scaling

We bring this analysis together by considering how the previous discussion of transport dynamics and hydraulics gives rise to a characteristic spatio-temporal scaling for braided rivers. Return to the condition that $F_r = 1$. This condition implies that

$$\frac{(x/t)^2}{2gh} = 1 \tag{22}$$

where x and t are the distance and time of fluid motion near the bed, respectively. Note first that h and L are proportional to each other. Then consider that if the total distance of transport of particles, $L \leq \zeta$, that $t \approx \tau$. If the maximum value of L (for the smallest d values) is equal to ζ, then $x = \zeta$ as well. Under the single condition, then, that $L_{\max} = \zeta$, we derive,

$$\frac{(x/t)^2}{2gh} \propto \frac{(L_{\max}/\tau)^2}{L_{\max}} \propto \frac{\zeta}{\tau^2} \propto 1 \tag{23}$$

Clearly, if $L_{\max} > \zeta$, and the particles get into another channel, the transport distance equation must be constructed with two terms, one for each channel. But this level of complication is beyond that of the present paper. In any case, it would seem that the further a channel is able to transport particles, the longer it should be able to extend.

6. Conclusions

The scaling of channel length ζ to channel lifetime, τ in braided streams is known to follow $\zeta \propto \tau^{-2}$. Constant Froude number implies from dimensional analysis that a generalized length over a time squared, $\Xi//T^2$, is a constant. We show that a probabilistic formulation for sediment transport is capable of providing the link between the Froude number scaling and the spatio-temporal scaling of channels in braided streams. The same theoretical framework has already been used to provide a basis for the condition $F_r = 1$. This theoretical framework also appears to provide a mechanism for the termination of individual channels due to their sensitivity to flow fluctuations. The theoretical results appear to be consistent with field observations. Further work needs to address explicit calculations of the probability that fluctuations in sedimentation can block channels of a given size at the appropriate frequency, as well as consider the dynamic role of bedforms in affecting flow resistance, thus accentuating the tendency for channels to maintain critical flow. In addition it will be necessary to determine whether theory and experiment require $F_r = 1$, or whether F_r near one is sufficient to generate the observed spatio-temporal scaling.

References

Chang, H.H., 1979. Minimum stream power and river channel patterns. J. Hydrol. 41, 303–327.

Chow, V.T., 1959. Open-channel Hydraulics. McGraw-Hill, New York.

De Vries, P.,2000. Scour in Low Gradient Gravel Bed Streams: Patterns, Processes and Implications for the Survival of Salmonid Embryos. PhD Dissertation, University of Washington, Seattle. 365pp.

Ergenzinger, P., 1987. Chaos and order: The channel geometry of gravel bed braided rivers. In: Ahnert, F. (Ed.), Geomorphological Models, Catena Suppl. 10. Catena Verlag, Cremlingen-Destedt, Germany, pp. 85–98.

Foufoula-Georgiou, E., Sapozhnikov, V., 2001. Scale invariance in the morphology and evolution of braided rivers. Math. Geol. 33, 273–291.

Grant, G.E., 1997. Critical flow constrains flow hydraulics in mobile-bed streams: a new hypothesis. Water Resour. Res. 33, 349–358.

Gupta, V.K., 2004. Emergence of statistical scaling floods on channel networks from complex runoff dynamics. Chaos, Solitons, and Fractals 19, 357–365.

Huang, H.Q., Nanson, G.C., 2000. Hydraulic geometry and maximum flow efficiency as products of the principle of least action. Earth Surface Processes and Landforms 25, 1–13.

Huang, H.Q., Chang, H.H., Nanson, G.C., 2003. Minimum energy as the general form of critical flow and for explaining variations in river channel patterns. Water Resour. Res. 2003;, 1–13. W04502, doi 10.1029/2003QWR002539.

Hunt, A.G., 1999. A probabilistic treatment of fluvial entrainment of cohesionless particles. J. Geophys. Res. 104, 15409–15413.

Hunt, A.G., Papanicolaou, A.N., 2003. Tests of predicted downstream transport of clasts in turbulent flow. Adv. Water Resour. 26, 1205–1211.

Inglis, C.C., 1947. Meanders and Their Bearing on River Training, Maritime and Waterways Engineers Division. The Institution of Civil Engineers, London.

Jaeger, C., 1956. Engineering Fluid Mechanics. Blackie and Son, London.

Kirkby, M.J., 1977. Maximum sediment transporting efficiency as a criterion for alluvial channels. In: Gregory, K.J. (Ed.), River Channel Changes. Wiley, Hoboken, NJ, pp. 950–967.

Knapp, D., 2002. PhD Dissertation, Washington State University.

Kolmogorov, A.N., 1991. Local structure of turbulence in incompressible fluid at very high Reynolds numbers (in Russian) Dok. Akad. Nauk. SSSR 30, 299–303, 1941. (English translation) Proc. Roy. Soc. Lond. A 434, 9–13.

Lamb, H., 1945. Hydrodynamics. Dover Publications, New York.

Reif, 1965. Fundamentals of Statistical and Thermal Physics. McGraw-Hill, New York.

Rodriguez-Iturbe, I., Rinaldo, A., 1997. Fractal River Basins: Chance and Self-Organization. Cambridge University Press, Cambridge.

Sapozhnikov, V., Foufoula-Georgiou, E., 1996. Do the current landscape evolution models show self-organized criticality?. Water Resour. Res. 32, 1109–1112.

Sapozhnikov, V.B., Foufoula-Georgiou, E., 1999. Horizontal and vertical self-organization of braided rivers toward a critical state. Water Resour. Res. 35, 843–851.

Schoklitz, A., 1937. Hydraulic Structure. Translated by Samuel Schulits, translation reviewed by L.G. Straub, Proceedings of American Society of Mechanical Engineering. p. 504

Tinkler, K.J., 1997a. Critical flow in rockbed streams with estimated values for Manning's n. Geomorphology 20, 147–164.

Tinkler, K.J., 1997b. Indirect velocity measurements from standing waves in rockbed rivers. J. Hydraul. Eng. 123, 918–921.

Topping, D.J., 1997. Physics of flow, sediment transport, hydraulic geometry, and channel geomorphic adjustment during flash floods in an ephemeral river, the Paria River, Utah and Arizona, Dissertation, University of Washington, 405pp.

Turcotte, D.L., 1997. Fractals and chaos in geology and geophysics, second ed. Cambridge University Press, Cambridge.

Turcotte, D.L., 2004. The relationship of fractals in geophysics to the new science. Chaos Solitons and Fractals 19, 255–258.

Vincent, K.R, Smith J.D., 2001. Eos Trans. AGU 82(47), Fall Meet. Suppl. Abstract H21C-0316.