BULLETIN OF THE GEOLOGICAL SOCIETY OF AMERICA
VOL. 59, PP. 463-512, 1 FIG. MAY 1948

CONCEPT OF THE GRADED RIVER

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ABSTRACT

Grade is a condition of equilibrium in streams as agents of transportation. The validity of the concept has been questioned, but it is indispensable in any genetic study of fluvial erosional features and deposits. This paper modifies and extends the theory of grade originally set forth by Gilbert and Davis.

A graded stream is one in which, over a period of years, slope is delicately adjusted to provide, with available discharge and the prevailing channel characteristics, just the velocity required for transportation of all of the load supplied from above. Slope usually decreases in a downvalley direction, but because discharge, channel characteristics, and load do not vary systematically along the stream, the graded profile is not a simple mathematical curve. Corrasive power and bed rock resistance to corrasion determine the slope of the ungraded profile, but have no direct influence on the graded profile. Chiefly because of a difference in rate of downvalley decrease in caliber of load, the aggrading profile differs in form from the graded profile; the aggrading profile is, and the graded profile is not, asymptotic with respect to a horizontal line passing through base level. It is critical in any analysis of stream profiles to recognize the difference in slope-controlling factors in parts of the overall profile that are (1) graded, (2) ungraded, and (3) aggrading.

A graded stream responds to a change in conditions in accordance with Le Chatelier's general law:—"if a stress is brought to bear on a system in equilibrium, a reaction occurs, displacing the equilibrium in a direction that tends to absorb the effect of the stress." Readjustment is effected primarily by appropriate modification of slope by upbuilding or downcutting, and only to a minor extent or not at all by concomitant changes in channel characteristics. Paired examples illustrate (1) the almost telegraphic rapidity with which the first phases of the reaction of a graded stream to a number of artificial changes are propagated upvalley and downvalley and, (2) the more or less complete readjustment that is effected over a period of thousands of years to analogous natural changes.

The engineer is necessarily concerned chiefly with short-term and quantitative aspects of the reaction of a graded stream to changes in control, while the attention of the geologist is usually focused on the long-term and genetic aspects of the stream's response to changes. But the basic problems are the same, and a pooling of ideas and data may enable the engineer to improve his long range planning of river control measures and permit the geologist to interpret, in quantitative terms, the deposits of ancient streams.

INTRODUCTION

The concept of grade, as a condition of equilibrium in streams as agents of transportation, has been the fundamental basis for the understanding of fluvial landforms for the last half century. The geologic literature contains, however, a number of markedly different definitions of the concept, and many geologists have been troubled by its defects and inconsistencies. An analysis of some of these difficulties leads Kesseli (1941) to conclude that the views of Gilbert (1877) and Davis (1902) regarding the equilibrium relationship are untenable and that the concept of grade must be abandoned. This article is an outgrowth of studies of stream planation surfaces in Wyoming (Mackin, 1936, 1937), was started several years before Kesseli's critique was published, and is a revision of the concept rather than a defense of the writings of Gilbert and Davis.

The engineering literature provides a counterpart for the concept of grade in the idea of the "adjusted" or "regime" condition in streams. The engineer is concerned primarily with short-term reactions of adjusted streams to damming, shortening, and deepening operations and other river training measures. The geologist sees erosional and depositional features in valleys as records of the long-term response of the graded stream to various natural changes in conditions controlling its activity. These natural changes in control are in many instances closely comparable with those introduced by man. Because they are a good test of the concept of the graded or adjusted condition, a number of paired examples of long- and short-term reactions of streams to analogous changes are brought together here; citations are drawn about equally from geologic and engineering writings.

There is much of common interest in this type of synthesis, but the geologist and the engineer differ widely in background and habits of thought, and an attempt to bridge the gap requires certain compromises in use of terms and manner of treatment. General policies are as follows:

(1) Future advances in knowledge of stream processes will certainly be based increasingly on quantitative measurement and mathematical analysis. But the quantitative aspects of transportation by running water are controversial and are not essential for an evaluation of the concept of grade; the treatment here is qualitative. If, by clarifying some of the genetic aspects of the problem in qualitative terms, or focusing attention on them, the article clears the way for more rapid quantitative advances, it will have served part of its purpose.

(2) There are two possible approaches to the study of streams as agents of transportation, (A) in terms of relationships between slope, discharge, channel form, and the size of grains comprising the load, and (B) in terms of energy transformations. Preferably, the two should not be combined. But they *are* combined in most of the papers cited, and, while the thesis of this article depends wholly on the first approach, some discussion of energy transformations is necessary. The manner in which the term energy is used is well established in the literature; it may be regarded by the specialist as loose, but he will be merely irritated rather than misled.

(3) Transporting power is considered to be a function of *velocity*, rather than the

depth-slope (tractive force) *relationship* that forms the basis for many mathematical treatments of transportation. This usage has the advantage of simplicity and, for present purposes, the differences are negligible (for analysis of these alternative theories see Rubey (1938) and discussion of Kramer (1935) by outstanding engineers, especially Matthes and Straub (p. 867–868).

(4) Partly in deference to the inveterate equation-skippers, but chiefly because critical differences between causes and effects do not appear in an equation, mathematical methods of expressing relationships are generally avoided.

ACKNOWLEDGMENTS

After the 15-year period during which the views outlined here were developed it is difficult for me to distinguish between ideas that were arrived at independently and those gleaned from reading, discussions with numerous geologists and engineers, and lectures in the classrooms of Douglas Johnson and W. M. Davis. An effort has been made to credit other workers with specific points made by them, but the 80-odd citations certainly do not cover all of the cases in which the same thoughts have been expressed before, especially in the writings of Baulig (1926), Davis (1902), Gilbert (1877, 1914), and Rubey (1933, 1938) among the geologists, and Lane (1937), Salisbury (1937), Schoklitsch (1937) and Sonderegger (1935) among the engineers. Early papers, chiefly of historic interest, are not included in the bibliography (in this connection see Baulig, 1926).

I am indebted to W. W. Rubey and Lee Stokes (U. S. Geological Survey), Stafford C. Happ and Allen S. Cary (U. S. Army Engineers), and Robert C. Hennes (Engineering, University of Washington) for critical comments on the manuscript.

VELOCITY AND LOAD

GENERAL STATEMENT

This section is a brief review of certain general principles of stream transportation, drawn chiefly from the works of Gilbert, Rubey and Hjulström. The principles are are based largely on laboratory studies and apply equally to graded streams and streams that are not graded. As outlined here they provide a basis for understanding observed behavior of graded streams; the concept of grade depends, not on any particular theory of transportation nor any special manner of apportioning energy losses, but on the form of the longitudinal profile developed by the debris-carrying stream under stable controlling conditions, and on profile changes that automatically readjust the stream to any change in controls.

ENERGY AND VELOCITY

The energy of a stream between any two points is proportional to the product of the mass and the total fall between the two points. This is, hereafter, the "total energy"; it increases with increase in discharge or slope but is increased also, negligibly for present purposes, by the presence of debris in motion in the water.

The energy is dissipated largely, or in some circumstances wholly, as heat developed by viscous shear within the stream. A rather artificial but useful distinction can be made between (1) energy dissipated in friction along the wetted perimeter of the channel (external frictional losses), (2) energy dissipated in friction between the diverse threads of the turbulent current (internal frictional losses), and (3) energy consumed in the transportation of load. The external and internal frictional losses occur whether or not the stream is engaged in transportation; these losses increase with increase in roughness of the channel, with irregularity in the trend or alignment of the channel, and with any departure from the ideal semicircular cross-sectional form that provides the shortest length of wetted perimeter per unit of cross-sectional area. Roughness, alignment, and cross-sectional form are referred to as "channel characteristics"; they determine the "hydraulic efficiency" of the channel. On the basis of an analysis of Gilbert's experimental results and other data, Rubey estimates that the frictional energy losses account for 96% to 97.5% of the total energy in some debris-carrying streams, and that the remaining energy is utilized in transportation (Rubey, 1933, p. 503). The point emphasized here is that the share of the total energy that is utilized in transportation is very small.

Transportation of boulders and pebbles that move only if they are rolled or dragged along the stream bed, and of smaller grains that must be lifted again and again by turbulent currents consumes energy. These pebbles and grains move slower than the water—the energy required to put them in motion and keep them in motion varies with grain size and quantity. Transportation of ultra-fine or colloidal particles with negligible settling velocities (in still water) does not tax the energy of the stream.

If the energy in a given segment were not utilized within that segment an acceleration in the rate of flow would result. Since this is usually not the case it appears that the energy in most segments is equal to the energy dissipated within those segments (Gilbert, 1877, p. 106). This conclusion taken together with the fact that the energy dissipated in internal and external friction is overwhelmingly greater than that consumed in transportation means that, total energy determined by slope and discharge remaining the same, relatively slight changes in the channel characteristics cause very marked changes in transporting power. The practical engineer concerned, for example, with design of non-silting and non-eroding canals is well aware of this relationship (Lane, 1937). It is not given due emphasis in geologic textbook discussion of stream transportation.

The several factors that enter into this energy balance in streams may be recast in terms of velocity:

Velocity increases with increase in slope of the water surface.

Increase in discharge is accompanied by increase in (a) the cross-sectional area and (b) the wetted perimeter of the channel. Since the natural channel is approximately rectangular in section the cross-sectional area increases approximately as the product of width and depth, while the wetted perimeter increases approximately as the sum of the width and twice the depth. Cross-sectional area therefore increases relatively to wetted perimeter with increase in discharge, and this change results in a relative decrease in frictional retardation of flow. Primarily for this reason, velocity varies with discharge.

As the channel departs from the ideal cross-sectional form, or as the floor and walls vary from smooth to rough, or as the trend varies from straight to tortuous, there is an increase in external frictional retardation of flow due to increased length of the wetted perimeter relative to cross-sectional area, and also an increase in internal frictional retardation of flow due to increased turbulence. For these reasons, velocity varies with variation in the channel characteristics.

Velocity is, then, a measure of the energy content of the stream. It varies with any change in the total energy resulting from change in slope or discharge, and, total energy remaining the same, it varies with any change in the energy dissipated in external or internal friction, as defined earlier. To complete the picture of energy-velocity interrelations for the case of the debris-laden stream we have Gilbert's experimental data indicating that velocity varies inversely with the amount of energy consumed in the transportation of load (1914, p. 225–230).

COMPETENCE

Competence is defined by Gilbert as a measure of the ability of the stream to transport debris in terms of particle size; the familiar statement is that the weight of the largest particles moved by a stream varies as the sixth power of the velocity. It is well known that there are notable differences in velocity in different parts of the cross section of a stream; the term, as used in the expression above, is usually interpreted as the average velocity. Rubey states that competence actually varies as the sixth power of the "bed velocity", and that the bed-velocity formula gives "reasonably close estimates of the maximum size of particles transported by some large natural streams for which adequate data are available" (Rubey, 1938, p. 137). For purposes of the present discussion, the significance of Rubey's analysis and of the experimental data presented by Gilbert and numerous other workers in this field is simply that velocity required for transportation of detritus increases very markedly with increase in particle size.

The "sixth-power law" was formulated to express the velocity requirements of that fraction of the load of a stream which moves by sliding, rolling, and bouncing along the stream's bed, *i.e.* the tractional load or bed load. But a large part of the load of most natural streams is transported in suspension. The size of the largest particles that can be carried in suspension depends, not on velocity directly, but upon the intensity of turbulence within the stream (Leighly, 1934). Turbulence itself is, however, a function of velocity among other factors; intensity of turbulence increases with increase in velocity. Thus, while Rubey's sixth-power law does not apply to the transportation of suspended load, the decrease in velocity requirements with decrease in grain size probably continues through the range of the larger grain sizes that are normally carried in suspension. A considerable fraction of the suspended load may consist of ultra-fine or colloidal particles with negligible setting velocities; maintenance of these ultra-fine materials in suspension depends only negligibly upon velocity.

CAPACITY

"Capacity", as defined by Gilbert (1914, p. 35), refers to "the maximum load a stream can carry". The experimental data on which the competence principle was based demonstrate also that, in a stream of given discharge, the velocity required for transportation varies with the quantity of any one grain size, the velocity requirements increasing with increase in the quantity or total weight of the material shed into the stream.

Gilbert used "capacity" in discussing data relating to transportation of weighed amounts of particles in the sand gravel size range under laboratory conditions. In spite of his explicit warning that his concept of capacity does not necessarily apply to natural streams (Gilbert, 1914, 223–230; *see also* Quirke, 1945) there has been a tendency so to apply it; the expressions "loaded to capacity", or "fully loaded", or "saturated with load" are frequently used in discussion of the graded condition in streams. These expressions are of course meaningless unless accompanied by some statement of the grain sizes or range in grade sizes which constitute the load. A stream "loaded to capacity" with coarse sand and pebbles could carry an enormously greater tonnage of material without change in velocity if the materials making up the load were crushed to silt size.

The capacity principle, like the competence principle, probably does not apply to the transportation of very small particles. Since the maintenance in suspension of ultra-fine clay particles and colloids (particles with negligible settling velocities) does not depend upon velocity, there is no theoretical upper limit to the amount of these materials that a stream of a given size and velocity can carry. A stream "loaded to capacity" with exceedingly fine particles would be a mud flow (Hjulström, 1935, p. 344–345).

There probably is in nature every gradation between normal streams and mud flows. Even the low concentrations of colloidal and ultra-fine particles that occur in normal streams undoubtedly tend slightly to increase carrying power by increasing the specific gravity of the water, and tend slightly to increase external and internal frictional energy losses by increasing the viscosity of the water. With higher and higher concentrations of these materials, particles of silt and sand and finally pebbles and boulders come to have negligible settling velocities in the medium until, in a mud flow, great blocks of rock can be carried buoyantly in a plastic mass that may move only a few feet an hour. These effects are negligible in normal streams.

THE TOTAL LOAD

Depending upon the lithologic characteristics, relief, and erosional processes in its drainage basin, and on processes in operation within the stream itself (as sorting), the range in grain sizes in the total load supplied to a given segment of a stream may vary widely. Moreover, the proportions of the several grain sizes in the total load may differ markedly in streams in which the range in grain size is the same. There is always, in normal streams, a decided "deficiency" in the supply of colloidal and ultra-fine materials.

Frequently the alluvial materials beneath and marginal to a stream channel include such an assortment of grain sizes that, as the velocity of the stream increases with seasonal increase in discharge, it is free to put in motion progressively coarser grain sizes up to the limits of its competence. There will be in this case a reasonably close relationship between the *quantity* of the debris in motion and the *largest grain* *sizes* that are in motion at a given time. Partly on this basis, and using an expression for the "average settling velocity of all of the debris particles being transported", Rubey has developed a means for evaluating both competence and capacity relationships in terms of bed velocity: "in a stream free to pick up much sand and gravel as its velocity is increased, the unit width load will vary roughly as the third power of the 'bed' velocity" (Rubey, 1938, p. 139).

Rubey points out that this third-power principle applies only approximately in even those streams to which a fair proportion of the several movable grain sizes (excepting the ultra-fines) are available. In this case both competence and capacity might be said to be a function of velocity. But a stream completely adjusted to the transport of a large amount of sand and silt may carry no pebbles at all, due to a deficiency in supply, although gravel sizes are well within its competence. Gilbert's experimental proof that the quantity of load increases as the grain size decreases suggests that in this special case the total load per unit width of stream varies as a power of the bed velocity higher than the third power; it might be said that "capacity" is the critical factor in this case. A stream may, on the other hand, be supplied with a load consisting predominantly of pebbles and boulders, with a notably small proportion of sand. It appears that in this circumstance the total load per unit width of channel may vary as a power of the bed velocity lower than the third power; competence might be said to be the critical factor. (For an example of the significance of this point in a practical problem of design *see* Whipple's discussion of Missouri River slope, 1942, p. 1191–1200, 1212–1214.)

These numerical values, as such, are not important for purposes of the discussion to follow. But the possibility of notable variations in the proportions of the several grain sizes making up the total load, and the bearing of these variations (qualitatively) on the velocity requirements in the transporting stream, are important. Hereafter the expression "increase (or decrease) in load" means increase (or decrease) in the quantity and average grain size, in accordance with the case treated by Rubey. The expression "increase (or decrease) in calibre of load" means increase (or decrease) in particle size, the total load remaining the same.

THE CONCEPT OF GRADE

A graded stream is not then, strictly speaking, one in which there is "a balance between total energy and the work given the stream to do", or in which "energy supplied equals energy consumed"; a non-accelerating flow of water carrying no load in a flume or a bed-rock channel fulfills these requirements, but would hardly be considered graded in the geologic sense. It is not a stream in which "slope is adjusted to load"; the carrying power of a stream is a function of velocity, and slope is only one of the factors which bear on velocity. One of the attributes of a graded stream is a "balance between erosion and deposition", but definition of the condition of grade in terms of this balance, and emphasis on the "constant shifting" of the balance, is unfortunate because it focuses attention on incidental short-term changes in the activity of the stream and loses sight of the long-term balance which is the distinctive characteristic of the stream at grade. A graded stream is not a stream "loaded to capacity" because streams never carry a capacity load (by Gilbert's definition). These definitions are partly or basically sound, but all of them include half-truths that are sources of confusion.

A graded stream is not in any sense a stream which is unable to abrade its bed because "all of its energy is used in transportation", or because "transporting the load requires all the energy that was formerly (during youth) applied to downcutting". The particles comprising the load are the tools used in abrasion, and since abrasion does not involve a dissipation of energy independent of that consumed in the propulsion of the tools, abrasion may be regarded as an incidental result of the bouncing, sliding and rolling motion of the particles.

A graded stream is one in which, over a period of years, slope is delicately adjusted to provide, with available discharge and with prevailing channel characteristics, just the velocity required for the transportation of the load supplied from the drainage basin. The graded stream is a system in equilibrium; its diagnostic characteristic is that any change in any of the controlling factors will cause a displacement of the equilibrium in a direction that will tend to absorb the effect of the change.

By *stream* we mean, of course, that particular segment with which we are directly concerned; many rivers have both graded and ungraded parts. The expression *over a period of years* rules out seasonal and other short-term fluctuations on the one hand and, on the other, the exceedingly slow changes that accompany the progress of the erosion cycle. *Load* and *discharge* deserve the prominence given in the definition not because they are the only or even necessarily the most important factors controlling slope, but because they are the only factors which are, *in origin*, wholly independent of the stream. *Slope* stands alone because it appears to be the only factor in the equilibrium which is automatically adjustable by the stream itself in such a direction as to accomodate changes in external controls that call for changes in velocity.

The balance involved in the condition of grade can be stated in an equation, but this method of expression is inadequate for present purposes because the terms of an equation are transposable. As set up in an equation, for example, load is a function of velocity. In answer to a query as to which is the cause and which is the effect, the average engineer will assert that velocity controls or determines the load that is carried by a stream; and he may have misgivings as to the sanity of the party who raised the question. In a flume or rock-floored torrent velocity does, in a sense, determine the load that can be carried. But, over a period of years, the load supplied to a stream is actually dependent, not on the velocity of the stream, but on the lithology, relief, vegetative cover, and erosional processes in operation in its drainage basin, and, in the graded stream, that particular slope is maintained which will provide just the velocity required to transport all of the supplied load. In this very real sense velocity is determined by, or adjusted to, the load. In the graded stream, load is a cause, and velocity is an effect: this relationship is not transposable.

The sections that follow approach the question raised by Kesseli as to the validity of the concept of grade by considering (1) typical examples of streams at grade, (2) factors that control the slope of the profile under stable conditions, and (3) reactions of graded streams to natural and artificial changes.

EXAMPLES OF STREAMS AT GRADE

A 50-mile segment of the Shoshone Valley east of Cody, Wyoming, contains a striking assemblage of river terraces, ranging from a few feet to several hundred feet above the stream. Inter-terrace scarps and the valleys of cross-cutting tributaries provide linear miles of exposures indicating that each terrace tread consists of a channeled and fluted rock floor, essentially flat in cross-valley profile, mantled by a uniformly thin (15 to 25 feet) sheet of alluvium. The rock floor of each terrace bevels inclined strata of varied types (Fig. 1). The mantle is made up largely of stream rounded pebbles wholly different lithologically from the local bedrock, and identical in composition with the detritus now being handled by the Shoshone River. The terrace surfaces and their planed rock floors exhibit smooth, concave-upward longitudinal profiles similar to that of the flood plain (Mackin, 1937, p. 825–837).

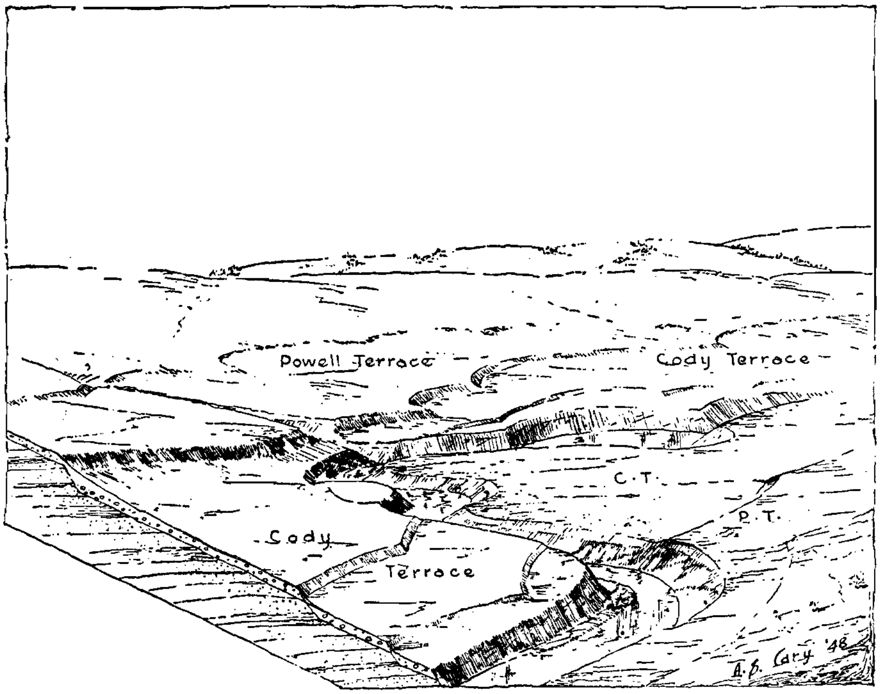

FIGURE 1.—*River terraces near Cody, Wyoming*

Drawn by Allen S. Cary from photographs taken from Cedar Mountain looking eastward down the Shoshone Valley. The alluvial veneer shown on the front of the block includes Shoshone River channel gravel and overbank silts, and side stream fan deposits; these materials are wholly different in origin and appearance but cannot be distinguished on the scale of the drawing. Note truncation of bed rock structure along the terrace scarps. The rear scarp of the Powell Terrace is about 90 feet high, but is largely covered by alluvial fans and slope wash.

The manner of origin of the terraces is indicated by the present activity of the stream, which is meandering on its valley floor. Some curves are slicing laterally at the base of vertical to overhanging rock walls, and some are shifting downvalley from similar cut banks. As each meander shifts it leaves behind a gravelly surface exposed at low-water stage—it is safe to infer that the thickness of this channel gravel is at least equal to the depth of the channel. It is evident that the gravel deposit grows by lateral accretion as the stream shifts, and if, as seems likely, it rests on a rock floor, then this floor must have been cut by the shifting stream, *pari passu* with the deposition of the gravel. The gravel sheet represents the bed load; it is soon covered by fine silt and sand representing the finer fractions of the suspended load deposited by slow-moving or ponded overbank waters, and later by slope wash and side-stream alluvial fans. We do not *know* that the valley floor gravel sheet rests on bedrock because we canot see its base. But the gravel sheet on each terrace does rest on beveled bedrock, and edges laterally against the base of scarps with the same systematic curvature in plan as those being cut by the stream (Fig. 1).

These relations, taken together, indicate that the terraces are remnants of valley floors cut in bedrock by the lateral planation of the Shoshone River during earlier periods of very slow downcutting or pauses in downcutting (Mackin, 1937). They are altogether different in origin and structure from the equally valid type of terrace formed by partial filling of a valley and later trenching of the fill. And they most certainly were not formed by incidental deposition of gravel (as by floods?) on surfaces produced by other erosional process (successive downstepping peneplanes?); in that they were formed by the same agency at the same time, the gravel veneers are related directly, not incidentally, to the planed surfaces on which they rest.

Individual terrace remnants in the Shoshone Valley are more than half a mile wide and a higher Shoshone valley floor (Pole Cat Bench) is over 2 miles wide and essentially flat in cross profile. In the opening out of valley floors of such great breadth the river must have shifted repeatedly from side to side, trimming back first one valley side and then the other. As indicated above, the stream is now engaged in the same activity on a valley floor with an average declivity of more than 30 feet per mile; the slope of the earlier valley floors is (and was) of the same order of magnitude. Since streams of similar discharge and channel characteristics are now vigorously cutting downward in rock with much lower slopes, the question arises as to what held in check the downcutting of the high-gradient Shoshone during the very long period of planation. Bedrock resistance can be readily ruled out as a controlling factor, for the valley is underlain by sandstones and shales, and such contrasts in resistance as do occur are not reflected in the profiles of the present stream or the terraces. Even more compelling is the fact that the river has repeatedly opened out very broad valley floors by lateral corrasion in the same bedrock during periods when its downcutting was negligible.

The Shoshone failed to trench its valley floor during the terrace-cutting stages, and the present stream fails to trench its present valley floor because its high gradient is perfectly adjusted to provide, with available discharge and with the prevailing channel characteristics, just the velocity required for transportation of a large load of coarse rock waste continuously supplied to it from ramifying headwaters in the rugged Absaroka Range. Adjacent ephemeral streams that head on the arid floor of the Bighorn Basin and are supplied only with fine-textured detritus maintain lower slopes than the master streams that head in the mountains, although their discharge is only a small fraction of that of the master streams (Mackin, 1936; see also Rich, 1935, and Hunt, 1946, for parallel cases in Utah). Water diverted from the Shoshone River and freed of bed load must be conducted down the terrace surfaces in concrete canals or in canals interrupted by concrete dams; it would otherwise entrench itself below the headgates of the laterals. The "velocity required for transportation" is such that the river rolls and bounces 8 to 12 inch boulders along its bed. The river bed is so efficient a grinding mill that boulders of such rock types as dense andesite are reduced in diameter by one-half within a few tens of miles. But the velocity requirements for transportation are so definitely fixed that the river, flowing over sandstone and shale, could not lower its slope by downcutting during the planation stages, and is not able to cut down at the present time. If the slope were altered, as by warping, the river would be forced to restore it by cutting or filling as the case might be. The Shoshone east of Cody was during the planation stages, and probably is at the present time, a typical graded stream.

Valleys of tributaries of the Columbia River system, particularly those of the Clark Fork and Spokane rivers, illustrate the same additional relationships even more strikingly than the Shoshone Valley. These Columbia Basin valleys were partly filled with glacial, glacio-fluvial, and glacio-lacustrine deposits during the Pleistocene and have since been partly re-excavated. Here again, we look to terrace remnants of higher valley floors because exposures in the dissected terraces supply morphological data that could be obtained from the present valley floor only by hundreds of borings. The postglacial stream terraces (not to be confused with a wide variety of other types of terraces produced during the period of ice occupancy) usually consist of a sheet of channel gravel, 10 to 30 feet thick, overlain by typical overbank silt plus loess and slopewash from higher valley sides. The gravel sheet rests on a channeled and fluted surface which truncates disordered structures in till, lake clays and silts, older gravels, and bedrock; deposition of the gravels accompanied the cutting of the surface on which they lie. The longitudinal slopes of the terraces and of the present valley floors range from 5 feet per mile upward and the larger common pebble sizes in the terrace gravels and the present river bars approximate 6 inches in diameter. It is useful to consider how rapidly, in so far as scouring power is concerned, large streams carrying coarse gravel on these high slopes could trench downward in silt, and at the same time to note that the streams opened out broad valley floors by lateral planation in silt and bed rock in adjoining segments of their valleys, without trenching. The high longitudinal slopes were maintained during the planation stages simply because these slopes were required to provide the velocity needed for transportation of detritus continuously supplied to the streams; for emphasis through hyperbole, one might say that they would have been so maintained had the subjacent materials been cream cheese.

These high-gradient streams were selected as examples to indicate at the outset that the term grade carries no connotation of low declivity. The low-gradient Illinois River is, as Rubey states, an excellent example of a stream in equilibrium (1931) but it is no more excellent than the Columbia tributaries, the Shoshone, the Mesa-stage Rock Creek (Montana) with a slope of about 90 feet per mile (Mackin, 1937, p. 848–850), or many Southwestern pediment streams with much higher slopes (Bryan, 1922). The classic examples are, of course, the wet-weather streams that carved Gilbert's planation surfaces around the flanks of the Henry Mountains in Utah (1877). If, as suggested by some engineering articles, the adjusted stream is one that is stable in channel form and position (Pickels, 1941, p. 166) then the (engineering) adjusted stream is only one special type of the (geologic) graded stream, which is stable only in slope.

The Columbia tributaries are useful also as examples because these streams are at many points locally superposed from the fill onto rock knobs and spurs, some of which have caused falls or rapids now and during earlier planation stages. These streams consist, in other words, of graded segments separated by segments that are not graded, but this circumstance is certainly no defect in the theory of grade.

THE GRADED STREAM AS A SYSTEM IN EQUILIBRIUM

The idea that a balanced or adjusted condition in streams is an expression of an equilibrium relationship, and that the graded profile is a slope of equilibrium is one of the oldest and most useful of geological concepts relating to streams. Many geologists seem to try to make their treatment of the balanced condition conform to the rigid definition of equilibrium used in the sciences of physics and chemistry. This point of view is reflected in the stock statement that the equilibrium is constantly shifting, approached, but rarely or never attained in the seasonally varying stream. It reaches its logical climax in Kesseli's argument that since discharge, velocity, and other factors are not literally constant in natural streams, no equilibrium can exist.

The requisite conditions for chemical equilibria (as between water and water vapor in a closed container) are; (1) absolute constancy of external controlling conditions (as temperature), and (2) a literally perfect balance between opposed tendencies (as the hail of molecules leaving and returning to the surface of the water). If the water-water vapor apparatus is housed in a laboratory where it is affected by constantly varying diurnal temperature changes, then, strictly speaking, the system rarely or never attains the perfect equivalence between opposed processes which is the essential mark of chemical equilibria. There is, moreover, in the precise chemical sense, a shifting between different states of equilibrium, but there is no such thing as a shifting equilibrium.

(1) *Constancy of controlling conditions* is certainly absent in any segment of a graded stream if attention is focused on its activity during any short period of time, as a year or part of a year. All natural streams vary in discharge, and in many the ratio of high-water to low-water discharge is several hundred to one. In some fully graded stream-transportation systems (in the geologic sense) there may be no discharge at all for most of the year. Velocity, load and all the other factors which enter into the economy of the balanced stream vary markedly with variations in discharge.

(2) *The perfect balance between opposed tendencies*, as an interchange between particles at rest on the bed and in motion in the stream, is not maintained in natural streams. In general, a stream flowing over alluvial materials within its competence tends to enlarge the channel during high-water stages, not only by increase in the height of the water surface, but also by scouring the bed. With decrease in discharge

and slackening in velocity as the high-water stage recedes, the stream deposits that part of the load which it is no longer able to carry. Indeed, the same change in controlling conditions may give rise to opposite changes in different parts of the same channel at the same time; increase in discharge and velocity usually causes, for instance, scouring on bends and filling on "crossings" in meandering streams (Straub, 1942, p. 619).

Kesseli, implying (1941, p. 580) that Davis was not aware of seasonal variations in discharge and velocity in natural streams, misses the point. Davis considered the stream as an agent of transportation over a period of years—he was concerned with the forest rather than the trees. Over a period of years sufficiently long to include all the vagaries of the stream, the two independent controls (discharge and supplied load) may be essentially constant. Whatever the conclusions from *a priori* reasoning as to whether constancy of these conditions *should be* maintained in nature, *a posteriori* reasoning based upon the existence of widespread corrasion surfaces of the type represented by the Shoshone terraces indicates that they *are* so maintained long enough to produce distinctive land forms.

Scouring and filling with seasonal fluctuations in discharge and velocity occur in all streams; it is the peculiar and distinctive characteristic of the graded stream that after hundreds or thousands of such short-period fluctuations, entailing an enormous total footage of scouring and filling, the stream shows no change in altitude or declivity. Here again the extensive stream-planed rock surfaces of the Shoshone Valley, with their thin veneers of alluvium, are a case in point. In this long-term sense, there is an equivalence of opposed tendencies in the graded stream.

The concept of equilibrium is the basis for modern quantitative treatments of stream transportation; Rubey's mathematical analysis of "capacity" is appropriately entitled *Equilibrium conditions in debris-laden streams* (1933). The origin and significance of slope variations in the longitudinal profile of the graded stream under stable conditions can be understood only in terms of equilibrium relations. In its sensitivity to change, and its tendency to readjust itself to the changed conditions, the graded stream exhibits the chief and diagnostic mark of a system in equilibrium. It is, in other words, useful and necessary to consider a graded stream as a system in equilibrium, and it is altogether proper so to consider it, provided that it is stated explicitly that the type of equilibrium is different in mechanism and detail from the types treated by the chemist and the physicist, and from the equally valid types recognized by the zoologist and the botanist.[1]

Recognition of these differences eliminates the need for apologetic statements, seemingly made in deference to the chemical usage, to the effect that the stream shifts with every short-period fluctuation from one state of equilibrium to another, or toward another which it never quite attains. To the extent that this view has been the vogue, Kesseli's statement that the condition of grade is "elusive" is amply justified. Distribution of discharge over a short period of time, whether essentially uniform or largely concentrated in rare floods, has much significance with regard to the charac-

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[1] For discussion of equilibrium as a "universal law" of wide application *see* W. D. Bancroft's Presidential Address to the American Chemical Society (1911). *See also* "The principle of dynamic equilibrium" as applied in Oceanography (Sverdrup *et al.*, 1942, p. 160).

teristics of the stream, and it is true that the greater part of the work of all streams both in erosion and transportation is accomplished during high-water stages. But whether the slope of the profile is determined during a brief annual period of high water (Baulig, 1926, p. 59) during the longer period of low water, or during some "bedforming stage" (Schaffernak, cited by Schoklitsch, 1937, p. 144) need not concern us here. It is not the particular stage of the stream in flood or low water, but the stream operating "over a period of years" that is the natural unit; the balance automatically maintained in this unit is, in its own way, quite as perfect as that of the most delicate equilibria dealt with in the "precise sciences".[2]

THE SHIFTING EQUILIBRIUM

The expression "over a period of years" was used advisedly in the statement above regarding the essential constancy of controlling conditions; all of the conditions are subject to change over a period of geologic time. The changes may be sudden, or they may occur at a rate corresponding with the slow progress of the erosion cycle. Other things being equal, the manner in which the stream responds to changes is determined by the rate at which they occur.

A once-graded stream may, in response to a change tending to cause downcutting, (1) lower itself so slowly that each of its slightly lower profiles is maintained in *approximate* adjustment to that phase of the slowly changing conditions in existence at the time of its formation or, (2) be transformed by a relatively sudden change into a wholly unadjusted series of waterfalls and rapids, and re-establish a graded profile at a lower level only after a considerable lapse of time.

Similarly, a graded stream may (3) respond to slow uplift of a barrier across its path by upbuilding, each of its successively higher profiles being in *approximate* adjustment to the conditions at the time of its formation. Rate of uplift of the barrier may, on the other hand, (4) so far outstrip the rate of filling that a lake basin is formed. The stream will in this case develop a new graded profile only after a period of delta building following cessation of uplift.

In cases (2) and (4) the streams were clearly ungraded or out of equilibrium during the transitional periods. The sharply contrasted condition of the streams in cases (1) and (3) may be thought of as representing a *shifting equilibrium.* Use of this expression to indicate maintenance of approximate adjustment to a long-term change in control is justified by the fact that it describes what actually occurs.

In the discussion to follow it will be necessary to return again and again to the contrast between processes in operation in the stream in which the condition of equilibrium is maintained, the stream in which the equilibrium is shifting, and the stream in which there is no semblance of equilibrium. The landforms and deposits

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[2] It is useful in this connection to look again at the water-water vapor system, this time focusing attention on the smallest conceivable surface area of water. It will be noted that *two* molecules may leave the water surface and only one return to it in any exceedingly short period of time. The level of the water surface is lowered. In the next unit of time two molecules return and only one leaves; the surface of the water returns to its former position. The point is that time and space relationships must enter into any consideration of equilibria. The time and space relations of the balance in graded streams are of a wholly different order of magnitude from those that obtain in the chemist's laboratory, but the perfection of equivalence of opposed tendencies is none the less perfect.

associated with these three conditions show differences that are of special significance to the geologist, but his terminology includes no simple and definitive terms for distinguishing them. It is therefore suggested, in accordance with Davis' (1902, p. 107) original proposal that "graded" be used specifically for the stream in which equilibrium is maintained, and that "degrading" and "aggrading" be restricted to cases of the shifting equilibrium. "Degrading" is downcutting approximately at grade, in contradistinction to such self-explanatory terms as trench or incise. "Aggrading" is upbuilding approximately at grade. "Regrading" refers to alteration in the form of the longitudinal profile by simultaneous aggrading and degrading in different parts (Johnson, 1932, p. 662). The term "degrade" is still available, of course, to describe the modeling of waste slopes in interstream areas.

There is no justification or need for using either "aggrade" or "degrade" to describe short-period variations in stream activity, that is, as synonyms for "filling" or "scouring" (of a channel), or for the more general terms "erosion" and "deposition". The following quotations from the last edition of an outstanding and most influential textbook illustrate, from the point of view of the present paper, a misuse of terms. The numbers in brackets are inserted for convenience in reference.

> "(1) As downcutting reduces the gradient . . . a time comes when the increasing burden of transporting the load requires all of the energy that was formerly applied to downcutting. . . . The long profile has become a *profile of equilibrium* and the stream is said to be *graded.* (2) When a part of a main stream reaches grade, the local tributaries soon become graded with respect to it. (3) Any change in gradient, discharge, or load would upset the graded condition by altering the rate of erosion. A flood, for example, might convert the graded stream into one actively degrading, but with subsidence of the flood the graded condition would be restored. (4) Again, great increases in load are known to have converted graded streams into actively aggrading ones; for example, when glaciers appearing in their headwater regions poured great additional quantities of rock waste into them". (Longwell, Knopf and Flint, 1939, p. 64–65).

The view expressed in (1) has been discussed earlier; downcutting, or abrasion in general, does not involve an expenditure of energy independent of that consumed in friction and transportation of load. In (2) the term grade is used in what the present writer regards the proper long-term sense, but in (3) it is used in a wholly different short-term sense. In (3) the term degrading is synonymous with deepening of the bed by scouring (this usage involves a situtation in which the *surface of the water is raised* when the stream is said to be degrading!). If a stream responds to a flood by degrading it presumably restores the graded condition by aggrading when the flood subsides. But in (4) aggrading is used to describe the response of a stream to a completely different long-term change in controlling conditions.

While no importance attaches to the terms, as such, it is the writer's opinion that the usage illustrated by these quotations is at least partly responsible for the confusion that forms the basis for Kesseli's attack on the theory of the graded river. The usage suggested here emphasizes the contrast between seasonal fluctuations in stream activity (or, indeed, the equally striking diurnal changes in certain proglacial streams) and such true shiftings of the equilibrium as those represented by epicycles of valley cutting and filling in the Southwest (Bryan, 1940; Bailey, 1935). The distinction is analagous to that made by the meteorologist between weather and climate, and it is just as fundamental. In addition, the proposed usage differentiates between the equilibrium that shifts in response to long-term change and the equilibrium that is maintained long enough to permit the stream to produce the distinctive landforms mentioned earlier. A question may arise as to the exact line of demarkation, in terms of feet of degradation or aggradation during so many thousands or millions of years, between the graded stream and the stream that is degrading or aggrading very slowly. Argument on this score leads nowhere—we classify natural phenomena not to assign each member of a series to a numerical pidgeonhole, but to clarify our understanding of their interrelationships. The distinction between the graded and the slowly degrading stream must, and should properly, depend on the nature of the problem and the point of view of the investigator.

FACTORS CONTROLLING THE SLOPE OF THE GRADED PROFILE

GENERAL STATEMENT

Longitudinal profiles of graded streams are often considered to be smooth, "concave upward" curves, that is, curves that decrease systematically in slope in a downvalley direction. Systematic downvalley decrease in slope is, however, by no means an essential or necessary attribute of the graded profile. The stream receives contributions of water and debris from every part of its drainage basin, but the additions are concentrated largely at tributary junctions and the ratio of water to debris varies markedly from place to place, from the high-water ratio of a tributary issuing from a lake or other natural settling basin to the high-debris ratio represented by a talus slide. Superposed on, and in part the result of, the changes in load and discharge are changes in the channel characteristics; these affect the hydraulic efficiency of the channel and hence the slope of the stream.

Such changes along its length might seem to count against considering the graded stream a system in equilibrium, or to indicate that it should be regarded as a type of shifting equilibrium. Indeed, comparison of the manner in which a stream accomodates itself to changes in control from segment to segment under stable conditions with its reaction to a change in conditions (as warping or a climatic change) is a useful mental exercise. But these two types of changes are completely different in origin, and it would be fatal to confuse them in analysis of a given longitudinal profile. While the velocity of each unit segment of a graded stream under stable conditions differs from that of adjoining segments (being kept in balance with local variations in velocity requirements by appropriate adjustments in slope) the close interdependence between all of the segments is such that they are parts of one well-defined system. Factors bearing on the slope of the longitudinal profile that is *maintained* without change as long as conditions remain the same are considered in this section.

DOWNVALLEY INCREASE IN DISCHARGE

It is a matter of observation that large graded streams usually have lower slopes than smaller graded streams. Similarly, a graded stream formed by the confluence of two graded streams usually has, below the junction, a slope lower than that of either of the confluents. The essential reason for these relations, mentioned earlier, is that with increase in size of the channel there is usually an increase in cross-sectional area relative to wetted perimeter and a consequent relative decrease in frictional retardation of flow. A result is that large streams commonly have higher velocities than smaller streams with the same slope, or, stated in terms of the profile of a single stream, a downvalley segment with large discharge can maintain a given velocity on a lower slope than an upvalley segment with small discharge. In other words, a mere downvalley increase in discharge requires (or permits) a corresponding downvalley decrease in the slope of the graded profile.

DOWNVALLEY INCREASE IN RATIO OF LOAD TO DISCHARGE

Trunk streams often head in regions of high relief and flow in their lower portions through regions of relatively low relief. Under these circumstances the ratio of total load to discharge in the contributions of tributaries may be larger in the lower than in the upper parts of the stream. For the same reason, the caliber of the load supplied to the stream by tributaries, slope-washing, creep, and talus fall commonly decreases from head to mouth. Downvalley decrease in total load relative to discharge and/or downvalley decrease in the caliber of load shed into the stream, to the extent that they occur, require a corresponding downvalley decrease in the slope of the graded profile.

DOWNVALLEY DECREASE IN RATIO OF LOAD TO DISCHARGE

The load of a graded stream may increase in a downvalley direction relative to its discharge because of evaporation or subsurface loss of water, or as a result of the entry of heavily loaded tributaries and various slower types of mass movement from its valley sides. Kesseli particularly emphasizes the latter process as being incompatible with the concept of grade, his statement being that if a stream be "fully loaded" it is manifestly impossible for it to acquire additional load as, for instance, the material caving from banks undercut in the process of valley floor widening (1941, p. 578). This statement is roughly equivalent to the contention that a given saturated solution, in the presence of excess of the solute, cannot take more of that substance into solution. The solution can and must, of course, become more concentrated if any change in control, as an increase in temperature, displaces the equilibrium in the proper direction. Similarly, the graded stream can accommodate itself to the transportation of increased load at any point; it usually does so by a local increase in declivity.

Steepening of the Missouri profile at and below the junction of the Platte River is a case in point. The average declivity of the Missouri for 31 miles above the mouth of the Platte was .74 feet per mile as measured in 1931, and the average slope for 44 miles below the junction was 1.24 feet per mile (Whipple, 1942, p. 1185—in the original report by Straub, cited by Whipple, slopes for unspecified distances above and below the junction are given as .68 and 1.16 feet per mile, respectively; Straub, 1935, p. 1145). The slope of the lower part of the Platte is 3.2 feet per mile. Steepening of the Missouri profile is ascribed by Straub and Whipple to entry of the heavy gravel bed load of the Platte into the Missouri, which carries chiefly sand and silt above the junction. On the basis of an extended study of Missouri slopes and load, Straub generalizes as follows: "As is to be expected, the steepest part of the Missouri River below the point of confluence of the Yellowstone is in the vicinity of the mouths of the tributaries adding the largest bed load" (Straub, 1935, p. 1145).

It should be emphasized that entry of the Platte gravels into the Missouri is not due to any recent change in conditions. The local steepening of the Missouri profile is *not* a matter of "deposition" of gravels by the Platte at its mouth because the Missouri is unable to carry the load. If this were so the streams would be upbuilding rapidly, which is not the case. The Missouri profile below the junction is just steep enough to permit the stream to carry *all* of the added load. The profile break has been and will be maintained without change as long as conditions remain the same, which is the same as saying that the Missouri is graded above and below the junction.

Additions to the load of a graded stream resulting from various types of mass movements of the type mentioned by Kesseli are accomodated in the same way. The fact that nearly all streams receive detritus from these sources and, nevertheless, usually maintain smooth concave-upward profiles past the individual caving banks means simply that these additions are usually so small, relative to the great bulk of rock waste in process of transport along the channel in any given period of time, that their effects on the longitudinal profile are usually lost to view in the general downvalley lowering of declivity resulting from the other changes discussed in this section.

Increased slope of the Missouri at and below the Platte junction is required to provide increased velocity needed for transportation of the increased bed load, but this is not the whole story. The Missouri, a meandering river above the Platte junction, is characterized at and below the junction by a broad irregular channel with numerous bars. This change in habits, a result of the added bed load, almost certainly increases frictional retardation and, hence, calls for a steepening in slope to permit development of any given velocity by the Missouri. In other words, the effect of the Platte is two-fold—the total steepening of the main stream profile represents the adjustment required to accomodate both the direct and the indirect effects of the influx of Platte gravels.

DOWNVALLEY DECREASE IN CALIBER OF LOAD

Graded profiles usually decrease in slope in a downvalley direction between tributary junctions, chiefly because of a downvalley decrease in caliber of load due to processes within the stream. The principle involved is that the velocity (and, other things being equal, the declivity) required for the transportation of the coarser fractions of a stream's load decreases with decrease in grain size, the total amount of the load remaining the same. The operation of this principle in natural streams is best illustrated by consideration of a graded segment without tributaries, in which additions and loss of water and detritus are negligible.

The lower portion of the Greybull River in the Bighorn Basin, Wyoming, approximates these ideal conditions. The river issues from the Absaroka Mountains and flows through the arid lowlands of the Basin to its junction with the Bighorn River, receiving its last perennial tributary (Wood River) about 50 miles above its mouth. A peculiar drainage pattern that delivers most of the intermittent drainage from immediately adjacent lowland areas to the Bighorn River by independent streams and an analysis of available discharge records provide reasonable assurance that there is no significant increase in the discharge of the Greybull in the 50-mile segment below Wood River. Throughout this segment the stream is meandering on a valley floor wider than the meander belt. Since the stream is neither aggrading nor degrading at a rate that would be appreciable over a period of years, the total load passing through all parts of the 50-mile segment in a given interval of time must be essentially the same, or may increase slightly downvalley as a result of bank erosion and other processes. In other words, variations in ratio of discharge to total load are of minor importance.

The profile of the Greybull valley floor decreases in slope from about 60 feet per mile at the head of the 50-mile segment to about 20 feet per mile at the lower end; that is, the profile is strongly "concave upward". Stream-cut rock terraces ranging from 50 to 1200 feet above the present stream show a similar eastward (downvalley) decrease in slope. The river was certainly in essentially perfect adjustment during the long periods of lateral planation recorded by the terraces and probably still is. The downvalley decrease in slope must in this case be ascribed largely to a decrease in caliber of load in transit; pebbles of the valley floor and terrace gravel sheets decrease notably in size in a downvalley direction (Mackin, 1937, p. 858–862).

Discussions of the form of the graded profile often neglect downvalley decrease in caliber of load as a control, or, what is worse, imply that this decrease is the *result* of the decrease in slope. Discussion of this point belongs in a later section; it is sufficient to state here that downvalley decrease in caliber of load is an important cause of downvalley decrease in the slope of the graded profile, and that, in the graded stream, the decrease in caliber is due primarily to attritional comminution of particles comprising the load.

The bearing of decrease in caliber of load due to attrition on the slope of the graded profile is particularly emphasized in the European literature. Schoklitsch, for example, cites Sternberg to the effect that certain central European rivers show a systematic decrease in the weight of particles comprising the bed load as a function of distance traveled. He then points out (1) that "An examination of the profiles of natural watercourses reveals the striking fact that, with few exceptions, the slope (like the size of the bed-sediment particles) decreases from source to mouth"; (2) that it is therefore "quite logical to attempt to ascertain a relation between this law [the Sternberg law of decrease in particle weight] and the shape of the profile, at least in the stretches in which the reduction in size of the particles is due to abrasion"; and finally, (3) that "study of a number of river profiles showed that the slope ... is proportional to the particle size". On this basis Schoklitsch develops an equation for the river profile (1937, p. 153). Many other writers have advanced the theory that graded or adjusted river profiles are mathematical curves, often without Schoklitsch's qualifications as to the effect of tributaries. The most recent contribution in this country (Shulits, 1941) presents a so-called "Rational equation of river-bed profile". The theory that longitudinal profiles closely approximate simple logarithmic curves has been utilized in geomorphic studies, chiefly in England. (*See*, for example, Jones, 1924; and Green, 1936; for critical discussion see Miller, 1939; and Lewis, 1945.)

Analysis of the mathematics of the graded profile and its important geomorphic implications lies beyond the scope of this article. The following will indicate how the theory applies in this qualitative treatment of the concept of grade:

Rubey, discussing the Shulits article, points out (1) that the Shulits equation is empirical rather than "rational"; (2) that many factors other than caliber of load bear on the slope of the profile, and (3) that the profile of a stream could vary directly as a power of bed-load diameters only if there were some "complex and as yet unformulated interrelationships among the many other variables" (p. 630).

The fact is, of course, that there can be no such interrelationships because there is no interdependence of all the factors bearing on slope in a trunk stream. The contribution of each tributary to a main stream is conditioned by the rocks, relief, and climate in the tributary drainage basin and is not systematically related to relationships in the main stream above the point where the tributary happens to enter. Changes in slope such as that caused by the Platte occur elsewhere on the Missouri, as indicated by Straub's generalization; in high-gradient western streams with which the writer is familiar these changes are even more striking than on the Missouri. The effect of tributaries is only one of several types of change along the length of the stream which are neither interdependent nor systematic. Mathematics can be an exceedingly useful tool in the study of river profiles, but it seems to the writer that the attitude that considers the job to be done because an *approximate* overall fit is obtained in the matching of curves is basically wrong; slurring over the irregularities gives a false impression of simplicity. Some types of numerical values, as the percentages of the several grain sizes in samples of sand and gravel, yield significant averages. But precise altitudes of points along a river profile are not subject to sampling errors; insofar as any understanding of its origin is concerned, the *breaks* in a semi-log plat of the profile are the significant elements, and a single straight line drawn through scattered points has little meaning. Downvalley decrease in caliber of load by reason of attrition is more nearly systematic than any of the other factors bearing on the slope of the graded profile. But caliber of load does not vary systematically in graded streams joined by tributaries, nor in graded streams in which the rock types in the load differ notably in resistance to attrition; even if it did, caliber of load is only one of a number of partly or wholly independent factors controlling the graded slope.[3]

RELATIONSHIP BETWEEN CHANNEL CHARACTERISTICS AND SLOPE

As indicated earlier, the hydraulic efficiency of a channel varies with the channel characteristics; self-evident theoretical considerations and experimental data establish this point so clearly that it needs no discussion here. Close relationship between efficiency, as determined by channel characteristics, and slope of the graded stream is demonstrated compellingly by the reduction in slope from about .96 to about .69 feet per mile in a few years, as a result of artificial smoothing of tortuous curves and narrowing, and consequent automatic deepening, of parts of the Missouri channel (Whipple, 1942, p. 1199). The overall length of the channel was not significantly changed by the channel improvement measures. In terms of velocity the essential reason for the reduction in slope is that a reduction in frictional retardation of the current in the corrected channel permits the stream to develop the velocity required for transportation of its load on a lowered slope. In terms of energy the

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[3] Slopes of ungraded stream segments are determined by the depth to which the stream has cut downward. In headwater basins in bed rock fairly uniform in resistance to corrasion the ungraded profile tends to decrease in declivity in a downvalley direction, the reason being downvalley increase in discharge and therefore corrasive power. Ungraded profiles of this type are superficially similar to graded profiles but they are completely different in origin; the ungraded profile is conditioned by the corrasive power of the stream, bed rock resistance to corrasion, and the length of time that the stream has been downcutting, while the graded profile is an adjusted slope of transportation that is influenced negligibly, if at all, by these factors. Mathematical analysis of a stream profile that fails to distinguish between graded and ungraded segments involves the fundamental error of mixing different types of data, and can lead only to frustration or to conclusions that are unsound.

explanation is that a reduction in "internal" and "external" energy losses in the improved channel causes an automatic adjustment of slope of such nature as to reduce the total energy of the stream, the energy utilized in transportation remaining the same.

Because different segments of graded streams vary widely in channel characteristics, these variations certainly bear on the overall form of the longitudinal profile. But before discussing these effects it is necessary to consider a fundamental question regarding the theory of grade.

THE CONCEPT OF ADJUSTMENT IN SECTION

The channel characteristics of a graded stream, like its slope, are developed by the stream itself. Both slope *and* channel characteristics vary from segment to segment, and any change in external controls usually results in changes in both of these variables. Because of the nature of his work the attention of the geologist is usually focused on slope; he knows, for example, that a graded stream responds to changes in load due to waxing and waning of glaciers in its drainage basin by appropriate adjustments in slope effected by upbuilding or downcutting. The attention of the engineer is, on the other hand, usually focused on the channel characteristics; he can, for example, enable an "adjusted" stream to transport an influx of mine waste that greatly increases its load by appropriate channel-improvement measures, without change in slope. A question arises, then, as to whether the foregoing definition which describes the graded stream as one "in which *slope* is delicately adjusted", etc., should not be revised to read, "in which *slope and channel characteristics* are delicately adjusted," etc.

That the question is a very real one is indicated by recent discussion of transportation by running water in the engineering literature which emphasizes especially the concept of the "adjusted", or "stable", or "regimen" (in the sense of equilibrium) cross-sectional form.[4] The discussion centers around problems of design of nonsilting and noneroding canals of various types. (Lane, 1937, with bibliography and discussion by 10 writers; important British papers include Griffith, 1927; and Lacey, 1930); but theoretical aspects of the relationship between cross-sectional form and transportation necessarily apply to natural streams. For example, Griffith observes (A) that natural streams with heavy bed load tend to flow in broad, shallow channels. He concludes (B) that the broad, shallow channel is the type of cross section best adapted for the transportation of heavy bed load. The general attitude of mind that makes (B) follow from (A) is expressed as follows: "A river fully charged with silt [meaning, in the geologic usage, debris without regard for particle size] *must obviously tend to adopt that form of section which will give it a maximum silt-carrying capacity*" (Griffith, 1927, p. 251) (italics mine).

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[4] I am indebted to W. W. Rubey for calling my attention to the need for somewhat more extended discussion of the concept of adjusted cross sections than was accorded that concept in an early draft of this article examined by him. He kindly loaned me an unpublished manuscript on the Hardin-Brussels quadrangles in Illinois, in which the channel characteristics of the Illinois River are analyzed mathematically (for published abstract, *see* Rubey, 1931a), and subsequent correspondence with him clarified and extended my own views. These views do not parallel his in all respects, and he is not responsible for them. But I would like to emphasize that everything that may have lasting value in this section is an outgrowth of his council.

There is perhaps deductive ground for believing that all the characteristics of natural channels, if they are to be "permanent", must somehow contribute to the ability of the stream to transport debris. It might be argued in the same deductive vein that the principle of "least work" would lead one to expect that changes in channel characteristics caused by a change in controlling conditions must be "adjustments" in that they must be of such nature as to adapt the stream to the new conditions. Analysis of the extent to which channel characteristics *are* adjustable in this sense may begin by considering channel-modeling processes in a straight channel in which movement of coarse-textured debris depends on velocity.

ADJUSTMENT IN SECTION IN THE STRAIGHT CHANNEL

The semicircular section that would provide the least frictional retardation of flow for clear water or for water carrying only ultra-fine or colloidal particles is rarely or never developed or maintained by flowing water charged with coarse debris for many reasons, the more important of which are: (1) a large bed load requires high bed velocity and widening by bank erosion that must continue until velocity at the banks is reduced to the point where the resistance to erosion of the bank-forming materials equals the erosive force applied to them. (2) Shoaling by deposition will accompany widening of the narrow channel by erosion because the particles of the bed load tend to lodge, and move, and lodge again, the velocity required each time to set them in motion being greater than that required to keep them in motion, and because a higher velocity is required to set in motion a particle on the bed than one on the sloping banks. The operation of these processes results in a channel that is, under different circumstances, semielliptical (Lacey, 1930, p. 273; Lacey in discussion of Lane, 1937, p. 160) or parabolic (Pettis, in discussion of Lane, 1937, p. 149–151) in section, and this section, once developed by automatic modification of an originally too narrow or too wide section, will be stable, or in adjustment, or regimen as long as conditions remain the same. But it does not follow that the channel so formed will necessarily provide the "maximum silt-carrying capacity."

Widening causes (1) reduction in bed load moved per unit width of bed by reason of decrease in velocity that accompanies decrease in depth, and, at the same time, (2) increases the length of the cross section (that is, the number of width units) through which the bed load is moved. Tendencies (1) and (2) are opposed insofar as transportation of bed load is concerned; especially because of (2) the cross-sectional form that provides maximum efficiency for transportation of debris is wider and shallower than the semicircular section that gives maximum efficiency for movement of water. The form of the cross section most efficient for transportation varies with slope, with amount and caliber of the load, and especially with the proportions of the total load that are carried in suspension and moved along the bed. These factors are partly interdependent, and they certainly influence the form of cross section that is developed and maintained by the stream. But the form of the cross section depends also, for reasons indicated above, on a factor wholly independent of the stream, namely, resistance of the banks to erosion. For any one set of slope-debris charge factors there will be one critical degree of erodibility of the bank-forming materials such that an originally too narrow channel will quickly develop a cross section with depth-width relations that provide "the maximum silt-carrying capacity". If the bank-forming materials are less erodible than this critical degree the stream may tend to develop the maximum efficiency section over a period of time; that is, the final stable channel section, however long delayed, may approximate the ideal form for the given slope-debris-charge factors. But if the bank-forming materials are more erodible than the critical degree the stream will adopt and maintain a section that is wider and shallower than the ideal transportation section. In the case of the high-slope stream carrying coarse gravel between banks of incoherent sand, widening may continue until the channel disintegrates into a plexus of split channels and gravel bars that is the antithesis of efficiency for transportation. In this case, and generally, the operation of the "least work" principle is merely a matter of expediency and compromise with local conditions—the river braids because an arrangement of minor channels and bars is somewhat less inefficient than a single exceedingly wide and uniformly shallow channel.

It appears, therefore, that a statement to the effect that a flow of water charged with debris must necessarily develop for itself that form of section that will give it a "maximum debris-carrying capacity" is an invalid generalization because it ignores erodibility of the banks; the influence of this factor increases with increase in slope and caliber of debris. The cross-sectional form developed by a stream may be "stable" in that it is not subject to modification as long as conditions remain the same, but "stability" in section is no guarantee of maximum efficiency for transportation. There is no need to labor this point with examples; it suffices to say that in many instances on record, efficiency for transportation in streams and canals has been greatly increased by artificial modification of self-adopted sections.

ADJUSTMENT IN SECTION IN THE SHIFTING CHANNEL

The tendency for erosional widening along both banks in the straight channel is, in the curving channel, localized and greatly accentuated at the outside of the curves. Whatever the type of meandering involved (Melton, 1936), the inner parts of the curving channel are bar-ridden shoals if the meanders are actively shifting. The meandering channel is usually deepest at the outside of the curves, but even here, if lateral shifting is rapid and the subjacent materials are resistant, the channel does not continue to operate in one place long enough to permit deepening to the potential depth of flood-stage scour. If outward shifting were stopped and if the shallows were filled so as to concentrate the flow in a narrowed channel, deepening would result. Efficiency for transportation would be further increased if the tortuous curves of the channel were smoothed.

The first point to be made, then, is that lateral shifting is one of the most important factors responsible for the inefficiency of the natural channel. This holds for the meandering stream, and it is true also for the braided stream. In general, efficiency for transportation varies inversely with the rate of lateral shifting.

The second point is that, other things being equal, the rate of lateral shifting in the graded stream increases with velocity and load; high-velocity, gravel-carrying streams on piedmont slopes of semiarid mountains shift laterally far more rapidly than low-velocity, silt-carrying streams of humid lowlands.

Consider now how a meandering stream will react to a change in controlling conditions, as for example, an increase in load. The engineer might be able to accommodate the stream to the increased load by artificially increasing the efficiency of the channel without increase in slope. Under natural conditions there will usually be both change in the channel characteristics and increase in slope. The increase in slope will be effected automatically by aggradation, and aggradation will continue until the slope is steep enough to provide the velocity required to transport, with available discharge and the prevailing channel characteristics, *whatever they may be*, all the debris delivered to the stream. Modification of the channel characteristics, considered separately as a mechanism of readjustment, will, on the other hand, be self baffling because increase in velocity required by increase in load will itself entail an increase in lateral shifting which will, in turn, tend to decrease the efficiency of the channel, and hence the velocity of the stream.

A graded stream may react to an increase in load by a more drastic change in the channel characteristics, namely, by a change from a meandering to a braided habit. Braiding involves the choking of each functional channel by bar building; the resulting maze of shifting minor channels has a total overall proportionate depth much smaller than that of the corresponding meandering channel. Here again, the effect on velocity of a change in the channel characteristics is precisely the reverse of that called for by the original change in external controls. Eventual readjustment to the new conditions (including increased load *and* notably decreased channel efficiency) will be achieved by increase in slope effected through aggradation.

These examples are certainly not intended to establish a general rule to the effect that, with a change in control, channel characteristics necessarily shift in a manner opposite to that required to bring the stream into balance with the new conditions. In some instances the effects of changes in the channel characteristics may be negligible, and in other instances they may contribute notably to the readjustment. But the examples do serve to bring out a basic difference in the role of adjustments in section and adjustments in slope in the equilibrium of grade; while adjustments in section may or may not accommodate the effect of a change in control, slope is always modified, by the stream itself, in such a manner as to absorb the effect of the stress. This relationship, the tendency for one of a number of partly interdependent variables to act as the outstanding counterbalance in effecting a readjustment to new conditions, is familiar in many types of equilibria. In the case of grade it means simply that the stream normally reacts to a change in controls calling for an increase or decrease in energy required for transportation by increasing or decreasing the total energy through modification of slope rather than by effecting economies in the energy dissipated in friction. A graded stream is a system, prodigiously wasteful of energy at every bend and shoal, kept in a constant state of balance under stable conditions, and brought back into balance after any change in controls, primarily by appropriate adjustments in slope.

EFFECT OF VARIATION IN CHANNEL CHARACTERISTICS ON THE GRADED PROFILE

It has been indicated that changes in ratio of load to discharge, in caliber of load, and other factors, occur in the graded stream, and that, while none of these changes is necessarily systematic, they usually combine to cause a downvalley decrease in the slope of the profile. Variation in the channel characteristics from segment to segment under stable conditions is due, in part directly, to changes in these other factors—for example, decrease in the caliber of the load that is moved along the stream bed tends, other things being equal, to be accompanied by an increase in proportionate depth. But changes in the channel characteristics are due largely to change in the resultant of these other factors, namely, velocity, and, in turn, rate of lateral shifting. The paragraphs below are intended to show the nature of the changes that may be expected in an ideal case and how these will affect slope.

In the upper parts of the graded stream,[5] velocity, and therefore the power of the stream to cut laterally, is high, but the valley floor does not greatly exceed the width of the stream itself, and the rate of lateral shifting is inhibited by confining rock walls. In the absence of well-developed meanders, the channel is relatively straight. For these reasons the actual rate of lateral shifting in any representative segment is relatively slow, and the proportionate depth relatively large.

The middle parts of the graded stream are characterized by fully developed meanders and by decreased velocity and therefore decreased corrasive power. But the actual rate of lateral shifting may be increased because the tendency for lateral shifting increases with decrease in the radius of curves, and especially because the stream now operates largely in unconsolidated alluvium on a wide valley floor. Other things being equal, proportionate depth decreases with increase in the rate of lateral shifting.

Finally, in the lower parts of the stream, velocity and corrasive power may decrease until there is a notable decrease in the rate of lateral shifting even in alluvial materials, with a corresponding increase in proportionate depth. Degree of sinuousity of the channel may remain the same (as in the middle parts) or may decrease.

It follows from the earlier discussion that, in the measure that these changes in the channel characteristics affect velocity, they will result in departures from the theoretical profile adjusted to discharge and load, but with uniform channel characteristics throughout. The effect will be to decrease the slope required to provide the velocity needed for the transportation of load with available discharge in the upper parts, to increase the slope required etc., in the middle parts, and to decrease the slope required etc., in the lower parts.

Deductions as to the effects on slope of the contrasts between the upper and middle sets of conditions are verified by relations described by Gilbert on the Yuba River in California:

> "Where the Yuba River passes from the Sierra Nevada to the broad Sacramento Valley its habit is rather abruptly changed. In the Narrows it is narrow and deep; a few miles downstream it has become wide and shallow. Its bed is of gravel, with slopes regulated by the river itself when in flood, and the same material composes the load it carries.

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[5] The terms "upper", "middle", and "lower" are used here for convenience to designate contrasted sets of conditions that bear on actual (not potential) rate of lateral shifting of a graded stream. These contrasted sets of conditions usually occur in the geographic order suggested by the terms, but depending on the geology of the drainage basin, they may occur in any order along the part of the stream that is graded. The terms should not be confused with the upper (downcutting), middle (cutting and filling), and lower (upbuilding) parts of the overall profile recognized by many workers. These subdivisions of the profile as a whole are discussed later; they have no place in the present treatment of factors bearing on the graded slope.

> "In the Narrows the form ratio during high flood is 0.06 and the slope is 0.10 percent. Two miles downstream the form ratio is 0.008 and the slope is 0.34 per cent. Thus the energy necessary to transport the load where the form ratio is 0.008 is more than three times that which suffices where the form ratio is 0.06; and it is evident that the larger ratio is much more efficient than the smaller" (Gilbert, 1914, p. 135).

Gilbert's "form ratio" is depth over width; it increases with increase in proportionate depth. While Gilbert does not say so, the abrupt change in depth-width relations on the Yuba is associated geographically with, and is certainly due in large part to, a change from a relatively low rate of lateral shifting in the rock-walled "Narrows" to a rapid rate of shifting on the piedmont alluvial plain. This downvalley *steepening* in slope at the point where a stream issues from a gorge is precisely the reverse of what we normally expect, and it is the reverse of what we usually find, because decrease in caliber of load, loss of water through infiltration and evaporation, and other factors, usually outweigh the effects of lateral shifting on channel characteristics and of channel characteristics on slope.

Factual relations at other California canyon mouths described by Sonderegger (1935, p. 296–300) indicate that the Yuba is not an isolated case; Sonderegger's reasoning as to the cause of the slope contrasts corresponds closely with that advanced here. These examples were not known to the writer when the deductions were set down in their present form—the examples therefore are "verification", by prediction of extraordinary or unique relationships, of the theory from which the deductions were drawn.

The effect of rate of lateral shifting on channel characteristics, and, in turn, of channel characteristics on slope in the middle and lower sets of conditions is strikingly illustrated by the Illinois River. This stream seems to exemplify the logical climax or end stage of the hypothetical sequence of downvalley changes in slope and channel characteristics outlined above.

> According to Rubey the lower Illinois channel does not shift perceptibly, is much narrower but somewhat deeper than that of the adjacent part of the Mississippi, and is deepest at the inside rather than at the outside of its bends. The river has a slope of less than 2 inches per mile, actually lower than that of the Mississippi from Memphis to the Gulf.
>
> An explanation of the remarkable habits of the Illinois, taken in part from Rubey's writings (Rubey, 1931a and the unpublished report mentioned earlier) and in part rationalized from factual relations described by him and shown on the topographic sheets, is as follows: The river flows in a valley formed by a much larger stream which served as the outlet of Lake Michigan during late-glacial times. Now, after several hundred feet of aggradation due in part to post-glacial aggradation of the Mississippi (which it enters), the lower Illinois is neither aggrading nor degrading at an appreciable rate. It is essentially graded (Rubey, 1931a, p. 366) and, as such, has a velocity adjusted to the transport of all of the debris shed into it. The slope of the stream is, in other words, adjustee to the present load under the prevaling conditions. The main stream is partly or wholly laked above the mouth of the Sangamon at Beardstown, and minor lateral tributaries below this point supply little coarse clastic debris. Under these circumstances the velocity required for transportation of the debris handled by the Illinois below Beardstown is very low—so low, in fact, that inertia fails to counteract the tendency of the principle current to follow the shortest and steepest route, which, in all curving streams, lies along the *inside* of the bends. This introduces a new factor tending to inhibit or halt lateral shifting (which would be very slow in any case because of low velocity) with the result that the stream maintains a relatively narrow and deep channel. The efficiency of this channel is so great, relative to that of actively shifting streams, that the Illinois develops the velocity required for the transportation of the load supplied to it on an exceptionally low slope.

The characteristics of the Illinois River are due largely to fortuitous circumstances (glacial drainage diversion, etc.), but the same type of regimen is approached in the lower flat reaches of many streams. Flattening of slope in the lower reaches is, for example, accompanied by a decrease in the rate of lateral shifting and an increase in proportionate depth on the Mississippi River (*see* Humphreys and Abbott, 1876, p. 107, 122) and the Brazos River in Texas (Barton, 1928, p. 622). It seems likely that this type of regimen, exceptional during the present epoch of crustal unrest and high-standing continents, may have been during earlier geologic periods a prevalent type in the lower parts of sluggish trunk streams draining areas of exceedingly low relief in the penultimate stages of the Davisian cycle of erosion.

LOCAL VARIATION FROM MEAN SLOPE

Detailed surveys based on reading of closely spaced gauges at the same river stage usually reveal what may be called local variations in the slopes of graded streams. These local departures from mean slope in any short segment may be fixed in position over a period of years, or they may shift along the stream; they usually vary in position with variation in discharge. They are in most cases clearly related to local variations in proportionate depth or detailed roughness, or to sharp bends, split channels, and other irregularities in trend which increase frictional retardation of flow. The fact that some of the local variations are obviously not associated with changes in load has led one student of streams to publish this rather remarkable nonsequitur: "the declivity of the adjusted stream is not a function of load". The fact is, of course, that declivity is not controlled by load alone.

These local variations in slope merit only brief mention here for the same reason that seasonal variations in discharge, velocity, and load were left largely out of account in the discussion of equilibrium relationships. Local changes in slope of the water surface, as such, are usually symptoms of some local "defect" in the channel; from the point of view of the geologist they are usually negligible because they are not reflected in the slopes of the valley floors produced by streams.

BACKWATER AND DRAW-DOWN EFFECTS

The usual downvalley decrease in the slope of the graded stream has led to the suggestion that the profile tends to be asymptotic with respect to a horizontal plane passing its base level. Similarly, the profile of a tributary is sometimes supposed to approach the slope of the main stream near the junction. It will be shown later that in a special case (aggradation) there is such a tendency. But it is the essence of the concept of grade that declivity is controlled by velocity or energy requirements and, in the graded stream, base level, as long as it does not change, has no bearing on velocity or energy requirements. Base level controls the *level* or *elevation* at which the profile is developed, but it does not influence the *slope* of the profile. For this reason a generalization to the effect that the graded profile approaches base level asymptotically is not valid.

In detail, profile relations in the vicinity of downvalley control points (that is, either general or local base levels) differ markedly (1) where the stream enters still water, (2) where the downvalley control is the lip of a waterfall, and (3) where a tributary enters a trunk stream. In (1) the lowermost part of the profile may show the "back-water effect" to a greater or less extent depending upon a number of factors; additional factors are involved if the "still water" is tidal, and/or if outbuilding of a delta is in progress. In (2) the lowermost part of the profile will show a "draw-down curve," that is, a steepening in the slope of the water surface resulting from a decrease in cross-sectional area due to acceleration in velocity toward the point of free fall. In (3) the backwater curve may affect either tributary or trunk stream or both depending upon their relative velocity and discharge and on the angle (in plan) between the streams at the confluence. These three different types of "base level" and their contrasted effects on a transitional zone in the lowermost part of the profile qualify the statement made in the last paragraph with respect to the relation of the overall profile to base level. But these relationships are local details having no direct bearing on the concept of grade.

EFFECTS OF DIFFERENCES IN ROCK RESISTANCE

Differential abrasion by streams tends to bring into relief differences in bedrock resistance; resistant rocks often form falls or rapids separating adjoining graded reaches. But if the stream be graded across the barrier, differences in bedrock resistance have no direct influence on its slope; theoretically and actually, as indicated earlier, graded streams cross belts of such contrasted rock types as quartzite and shale without change in slope at the contacts. Differences in the rock types traversed by the graded stream may, of course, cause changes in slope if associated contrasts in topography or lithology alter the amount and caliber of the load supplied to the stream, or if associated contrasts in valley-floor width affect rate of lateral shifting or details of trend or cross-sectional shape of the channel.

SUMMARY

The longitudinal profile of a graded stream may be thought of as consisting of a number of segments, each differing from those that adjoin it but all closely related parts of one system. Definition of the unit segment (in terms of length, permissible slope variation, etc.) depends on the purpose of the investigation.

Each segment has the slope that will provide the velocity required for transportation of all of the load supplied to it from above, and this slope is maintained without change as long as controlling conditions remain the same. The graded profile is a slope of transportation; it is influenced directly neither by the corrasive power of the stream nor bed rock resistance to corrasion.

Some changes from segment to segment in factors controlling the slope of the graded profile are matters of geographic circumstance that are not systematic in any way; these include the downvalley increase in discharge, and the downvalley decrease in load relative to discharge, that characterize trunk streams flowing from highland areas through humid lowlands. Other changes, as downvalley decrease in caliber of load by reason of attrition, may be more or less systematic between tributary junctions. Still others, as change in channel characteristics, are partly dependent on changes in load and discharge; the channel characteristics are determined chiefly by caliber of load and rate of lateral shifting of the channel, and the rate of channel shifting is itself dependent on velocity and erodibility of the banks.

These changes are usually such as to decrease slope requirements in a downvalley direction but, because none of them is systematic, the graded profile cannot be a simple mathematical curve in anything more than a loose or superficial sense. We can proceed toward an understanding of the graded profile, not by "curve matching", but by rigorous analysis of adequate sets of data for unit segments of natural and laboratory streams numerous and varied enough to reveal the effect of variation of each of the factors separately. An essential prerequisite for efficiency in the gathering and analysis of the data is recognition of the difference between the graded profile that is maintained without change, and the ungraded profile that is being modified by upbuilding or downcutting.

RESPONSE OF THE GRADED STREAM TO CHANGES IN CONTROL

GENERAL STATEMENT

The response of a graded stream to any change in control is systematic in that it is predictable in terms of Le Chatelier's general law: "If any stress is brought to bear on a system in equilibrium, a reaction occurs, displacing the equilibrium in a direction which tends to absorb the effect of the stress." This section outlines the manner in which a graded stream, as a system in equilibrium, reacts to "stresses" by considering the nature of its response to changes in some of the controlling conditions which were, in the preceding section, held constant.

The method of presentation adopted to some extent above but used more particularly here involves deduction, from the general concept, of specific reactions that should be expected as results of a number of changes in control, and the matching of these "expected consequences" with field examples. This method of testing the theory is neither superior nor inferior to the laboratory model method; it is simply different from the experimental method which it supplements but certainly cannot replace. Its validity as a test depends on (1) whether the deductions are logical, (2) whether the effects are specifically related to the stated causes, and (3) whether the examples are representative. Obviously, in some cases, the reasoning is inductive; the deductive method of presentation (Johnson, 1940) is not followed rigorously in most of these instances. But it is worth nothing that many of the reactions were in fact predicted purely on the basis of deduction, and later verified by reference to the record, and that a survey of factual relations set forth in the literature has failed to discover any effect of a given cause that does not fit the concept.

Examples are not cited, or are mentioned briefly, where relationships are clear cut or generally familiar. In some instances scores of examples bear out the deductive analysis, each differing from the others in nice detail; some selection was therefore necessary. In general, examples were selected in which a given cause can be related to a definite effect with the least explanatory argument. If alternative examples occur to the reader that illustrate a point more clearly than those cited, that is good; if, on the other hand, there are cases that fail to conform to the general thesis a description of them will be a contribution to our understanding of streams.

Engineering examples are usually intended to show the almost telegraphic rapidity with which preliminary reactions are propagated upvalley and/or downvalley from the point where a change has been introduced by man; some of the geologic examples show the nature of the response to analogous natural changes over a period of time in which more or less complete readjustment may be attained.

Discussion in the paragraphs below is confined to the general mechanism by which readjustment is effected. Contrasted methods of aggradation, the bearing of certain secondary cause-and-effect couples on the slope of the final readjusted profile, and the contrast between the form of the adjusted (or readjusted) profile of the graded stream and the disadjusted profile of the aggrading or degrading stream are treated later.

INCREASE IN LOAD

A once-graded stream responds to an increase in load primarily by steepening its declivity below the point of influx. The steepening is accomplished by deposition of part of the excess load in the channel at the point of influx with a consequent upbuilding of the channel at that point and the formation of a steepened part immediately below.[6] Steepening of any segment permits increased transport of load through that segment to the next segment which is in turn the site of deposition and steepening. Thus the effect of an increase in load is registered by the downvalley movement of a wave of deposition, large or small depending on the rate and manner of addition of the load, and deposition must continue throughout the stream below the point of influx until the slope is everywhere adjusted to the transport of all of the debris delivered to it.

The classic example of marked and immediate response of streams to increase in load associated with works of man is the aggradation and resulting widespread destruction of agricultural lands along the eastern side of the Great Valley of California caused by hydraulic mining on the western slopes of the Sierra Nevada between 1855 and 1884, when court decisions halted discharge of mining debris into the Sierra streams (Gilbert, 1917). Various surveys to 1894 are summarized as follows:

> "the deposit . . . was 20 miles long, had a maximum width of three miles, covering 16,000 acres and containing 600,000,000 cu. yd. It was 20 feet deep at the river's mouth, 35 feet deep at the edge of the foothills, and 80 feet deep 5 miles higher up on the Yuba River. The grade of the original bed was 5 feet per mile. After the fill was made, the grade per mile was 2½ feet at the mouth, 10 feet at the middle zone, and 20 feet on the upper reaches" (Waggoner, in discussion of Stevens, 1936, p. 271).

The effect of increase in load due to natural causes is most strikingly exemplified by the aggradation that commonly occurs when river valleys are invaded by glaciers. Automatic steepening of the declivity of the proglacial stream is not necessarily proof that great additional *quantities* of debris are being shed into them. The detritus carried from upvalley and delivered to the stream at the terminus of the glacier is usually much coarser than that formerly delivered to the same point from upvalley by running water, and increase in caliber may be more important than increase in quantity of load as a cause of the profile steepening.

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[6] A stream affected by a change in controls of any type "deposits part of its load" or "picks up more load" by appropriate modifications in the amount of material deposited and picked up in the course of its normal seasonal fluctuations, and the net differences are usually very small compared with the great bulk of material moved and relaid during these fluctuations.

Because glaciation is usually a relatively brief episode in the life history of the drainage basin, with the main cycle of advance and recession interrupted by numerous minor pulsations, and with continual change in topographic relations governing discharge of water and debris from the ice front, proglacial streams rarely or never attain adjustment. Their profiles therefore differ from the more or less completely adjusted pre-glacial and post-glacial profiles not only in overall slope, but also in form. (*See*, for example, MacClintock, 1922, p. 575, 681.)

DECREASE IN LOAD

A decrease in load may be thought of as occurring at any point in the stream. It may be stated for the time being that the stream simply makes up for deficiency in the load supplied from above by picking up additional load from its channel floor. The net result is downcutting, with a consequent lowering in declivity downvalley from the point where the change occurred. Downcutting must continue until the profile is reduced to that slope which will provide just the velocity required to transport the reduced load.

Because of the operation of the reservoir as a settling basin, the dam is the most common manmade cause of decrease in load. The downcutting that may result is typically shown by the Rio Grande below the Elephant Butte Reservoir and the Colorado below Lake Mead (Stevens, 1938) the Saalach River below the Reichenhall Reservoir (Schoklitsch, 1937, p. 157) and in many other cases (Lane and others, 1934). This effect of decrease in load due to damming is almost always complicated by elimination of peak discharges and velocities that results from the use of the reservoir as a water storage basin.

Terraces cut in earlier fill characterize Pleistocene outwash plains and valley trains. The usual downvalley convergence of terrace profiles cut during the period of deglaciation is probably due primarily to decrease in caliber of load reaching any given segment of the degrading stream as the distance between that segment and the receding ice front increases.

Davis (1902, p. 261) treated the very gradual decrease in stream slope that results from decrease in load due to reduction in relief during the humid erosion cycle; Johnson (1932) and others described analogous effects around the borders of shrinking desert ranges.

CHANGES IN DISCHARGE

If a segment of a graded stream receives all or most of its load at the upper end, changes in discharge call for readjustments in the slope in much the same way as changes in load. A decrease in discharge requires an increase in declivity because the load, remaining the same, must move faster through a smaller cross section, and because, as indicated earlier, decrease in the cross-sectional area of the channel involves a relative increase in frictional retardation of flow and, hence, a decrease in velocity. The stream affected by a decrease in discharge, being unable to transport all the load supplied to it on its former slope, deposits some of the load and thereby steepens the slope, the process continuing until the reduced stream is able, by reason of increased velocity on the steepened slope, to transport all the load shed into it. The opposite adjustment occurs in the case of increased discharge.

A special case of the operation of this principle is described by Salisbury (1937) in the lower Mississippi Valley. Subsequent to diversion of part of the Mississippi discharge into the Atchafalaya channel in 1882 there has been, downvalley from the point of diversion, silting of the bed of the reduced Mississippi, and lowering of the slope of the augmented Atchafalaya. Clearing of rafts on the Red River (the upper Atchafalaya), confinement of both the Mississippi and the Atchafalaya between levees, and other works of man have altered the regimen of both streams since the diversion. For these reasons, and particularly because of the exceedingly low slopes of the streams involved, the effects of the diversion are revealed only by careful evaluation of the evidence by Salisbury (chiefly in terms of variation in gauge heights) and by Lane (in discussion of Salisbury's paper, chiefly in terms of variation in discharge).

The distance from the point of diversion to the Gulf is about 125 miles along the Atchafalaya, and about 310 miles along the Mississippi. Deterioration of the discharge capacity of the Mississippi channel since the diversion began and concomitant increase in discharge through the steeper Atchafalaya suggest that we are viewing a type of deltaic drainage change, set in motion in this instance by man, which must have occurred repeatedly in the past under natural conditions. Salisbury's demonstration of silting in the trunk channel below the diversion provides an explanation of a mechanism by which deep-channel, slow-shifting streams like the Mississippi may transfer themselves to different radial positions as delta growth proceeds.

The principle and the general mechanics of readjustment are precisely the same in the more general case of a main stream which receives notable additions of debris from tributaries, but the effects on the form of the profile may be very different because loss of discharge in the main stream calls for local readjustments of slope at each tributary junction, usually enough steepening to permit the reduced main stream to transport the load supplied by the tributaries.

Aggradation of the trunk stream at and below tributary junctions is now in progress on the Rio Grande and the Colorado River below the Elephant and Boulder dams, due chiefly to elimination of peak discharges that formerly moved detritus delivered to the main channel by flash floods on the tributaries (Stevens, 1938). In these cases the effect of reduction of peak discharges locally outweighs the general tendency for downcutting due to retention in the reservoir of the bed load supplied from the upper parts of the trunk stream.

Recent widespread incision of valley floors by streams in the Southwest is ascribed by Bryan (1925; 1940) and Bailey (1935) to increase in peak discharge resulting from an increased rate of runoff due to reduction in vegetative cover. Assuming that this diagnosis is correct, whether the cause be climatic change (Bryan) or overgrazing (Bailey) the recent arroyo cutting is of special interest because, as in the examples last mentioned, a change in distribution of discharge through the year produces effects similar to those caused by a change in discharge, and because the downcutting, a preliminary result of deterioration in vegetative cover, may revert to upbuilding when and if increased load resulting from accelerated erosion of the denuded slopes begins to affect the streams.

The most common geologic cause of decrease in discharge is drainage diversion, and the most evident effect is the growth of side-stream fans in the valley of the reduced stream. The classic case is the partial blocking of the valley of the Petit Morin after capture of its headwaters by the Marne and the Aube; the marsh of St. Gond was caused by detrital accumulations which locally reversed the slope of the valley of the Petit Morin (Davis, 1896, p. 603, 604). Local slope reversals on a larger scale are represented by Lake Traverse and other lakes in the valley now occupied by the Minnesota River; this small postglacial stream has been unable to maintain the low-gradient valley cut during the Pleistocene by the very much larger "Warren River", which served as the outlet of Glacial Lake Agassiz. The final step in the blocking process is illustrated by an enormous accumulation of tributary fan detritus, possibly as much as 700 feet thick, in the gap cut by the former Shoshone River through the Pryor Mountains in Montana; in this case the discharge of the Shoshone was so greatly reduced by headwater diversion that the direction of flow of the beheaded trunk stream was completely reversed by tributary fans (Mackin, 1937, Fig. 6).

It appears, therefore, that reduction in discharge in a trunk stream joined by tributaries tends in general to cause profile changes that express the increased relative importance of the tributaries in the economy of the drainage system, and that, depending on how drastic is the reduction of the trunk stream, and how vigorous the tributaries, the main stream may respond by appropriate modification of its profile during a period of disadjustment or may disintegrate into a series of lakes and reversed segments.

RISE OF BASE LEVEL

A rise of base level is equivalent to the rising of a barrier across the path of the graded stream. Each unit of increase in the height of the barrier tends to flatten the declivity immediately upstream. The stream, unable because of decreased declivity to carry all of the load through the flattened segment, deposits in the segment, thus increasing the declivity and transferring the flattening upvalley. Continuation of the process results in upstream propagation of a wave or, better, of an infinite number of small waves of deposition.

If the barrier is raised slowly the stream may maintain itself in approximate adjustment during the process; its rate of aggradation is determined by the rate at which the barrier is elevated. If the barrier is raised rapidly or instantaneously a lake is formed; the distal part of the delta is then the "flattened part" of the profile, and the rate of aggradation is determined by the rate of delta building into the lake. In either case the successive profiles developed during the period of readjustment will differ markedly in form from the original profile and from the eventual completely readjusted profile. The final readjusted profile will tend toward parallelsim with the original profile, differing in this respect from the cases treated above. But, because of secondary effects of aggradation to be discussed later, precise parallelism will usually not be achieved; the only generalization that can be made is that the new profile will be everywhere adjusted to the new prevailing conditions.

Rapid upvalley propagation of a wave of deposition due to rise in base level is well shown by profile modifications brought about within a few years by erection of Debris Barrier #1 on the Yuba River (Gilbert, 1917, p. 52–63); Gilbert's figure 7 illustrates the typical wedging out upstream of the preliminary detrital accumulations. (See also Sonderegger, 1935, Fig. 2, p. 298).

The same effect, but on a much larger scale, is seen in aggradation on the Rio Grande above the Elephant Butte Reservoir (Eakin and Brown, 1939, p. 90–99). At San Marcial, a town near the head of the reservoir, the channel of the Rio Grande has been raised at least 10 feet since 1916, when the reservoir was completed, and the town site is largely buried in silt. Blaney states that surveys made in 1934 show a rise of the channel of 7 feet since 1918 at La Joya, and 2 to 4 feet at Albuquerque (Blaney, in discussion of Stevens, 1936, p. 266). La Joya and Albuquerque are about 50 and 100 miles (airline) above San Marcial, respectively, and Albuquerque is 500 feet above the level of the reservoir.[7]

The Elephant Butte Reservoir is about 40 miles long and the spillway crest is 193 feet above the original river bed. A statement to the effect that the eventual readjusted profile of the Rio Grande, long after complete filling of the reservoir, will be parallel with the original river bed and about 200 feet higher is, of course, wholly indefensible. It is, however, probably closer to the truth than the comfortable assumption that the final debris fill will wedge out to zero within a few tens of miles above the original head of the reservoir. The profile of the aggrading Rio Grande during the period of disadjustment will be less steep than the original profile, but the final readjusted profile may be less steep or steeper. It is not necessary to look so far into the future for trouble—a few tens of feet of upbuilding to the latitude of Albuquerque would destroy highways and railways, towns, irrigation works, and farmlands with an aggregate value that may exceed the cost of the dam.

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[7] Stafford C. Happ points out (personal communication) that increased sedimentary loads of tributaries in the last 50 years have probably caused aggradation in the Rio Grande Valley independently of the influence of the Elephant Butte Reservoir. If quantitative evaluation of adequate data indicate that all of the upbuilding noted by Blaney at La Joya and Albuquerque is due to these "upvalley" influences it will mean simply that the effects of the reservoir have not yet reached these points. For definite evidence that aggradation had extended to a point at least 15 miles above the reservoir by 1941 see Happ, in discussion of Stevens (1945, p. 1298).

Aggradation of the Kickapoo River in Wisconsin is ascribed by Thwaites and Bates chiefly to Pleistocene upbuilding of the Wisconsin River, which it enters; it is in effect, therefore, a case of aggradation caused by a "geologic" rise of base level. As in all such cases, aggradation undoubtedly altered conditions controlling slope on the Kickapoo; it is interesting, nevertheless, to note that the modern profile for the first 100 miles above the mouth corresponds closely in slope with the original profile determined by well logs (Bates, 1939; Thwaites, 1928, p. 628).

In some respects analogous to the more or less adjusted Kickapoo and serving especially as an antidote for any idea that the readjusted stream *must* be less steep than the original stream is the striking case of aggradation above a debris barrier on the Bear River in California; the approximately adjusted profile of this river above the barrier is notably steeper than the original profile of the stream (Stevens, 1936, p. 219).

Johnson and Minaker (1945, p. 904) cite an unpublished report by Kaetz and Rich to the effect that study of profiles above 22 debris barriers shows that the slopes of the deposits average from 37 to 49 per cent of the original slope—these cases are recognized as not having reached "equilibrium conditions". They state further that in model studies slopes up to 90 per cent of the original streambed slopes were obtained. Their interpretation of the difference in the form of the original profile and the profiles developed during aggradation differs in detail from those advanced later in the present article, but their factual data, reasoning, and conclusions conform in all respects with the general theory of grade.

Loss of storage capacity by silting is one of the outstanding engineering problems of the century; the problem is many-sided, and the literature is extensive. (*See*, for example, Brown, 1944; Stevens, 1936, 1945; Witzig, 1944; and papers cited therein.) Two practical implications of the theory of grade bearing on special aspects of the problem follow so directly from the views and factual relations cited above as to merit brief mention in passing.

It seems to be common practice to determine the "useful life" of a reservoir (usually the time required for filling with debris to spillway level at the dam, but varying depending on the purpose of the reservoir) by dividing the yearly increment of debris into the capacity of the reservoir for water, with due allowance for compaction and related factors. This method is evidently based on the tacit assumption that, when the delta front has advanced any considerable distance toward the dam, the river will carry its detrital load from the original head of the reservoir to the delta front on a surface of no slope (i.e., the water level). Mere statement of this basic assumption in these terms demonstrates its absurdity. Remedial measures costing large sums are in part contingent on predicted rates of storage depletion; these predictions, particularly on high gradient streams carrying a coarse bed load, are subject to gross errors when they fail to take into account the progressive increase in the proportion of the stream's load that is deposited on the valley floor above the reservoir as aggradation proceeds. (*See* Eakin and Brown, 1939, p. 6, 7; Happ, in discussion of Stevens, 1945, p. 1298–1300.) Research on what may be called "the form of the aggrading profile" is much needed, and the results must of course be quantitative to be useful. It is the writer's conviction that some qualitative understanding of the complex interrelationships of the factors at play in aggrading streams is the essential prerequisite for such quantitative studies if they are to yield general "laws" rather than a set of empirical constants of the type that lead some engineers to conclude that each individual stream has its own rules of conduct.

Study of the form of the aggrading profile may be expected to pay dividends not only by increasing accuracy of the storage depletion rate but also by developing methods by which the rate may be decreased. As indicated especially by the soil conservationists (Brown, 1944), a basic part of any comprehensive attack on the silting problem is reduction insofar as practicable of the debris shed into streams from uplands in the drainage area. The next step is to decrease the load delivered by the streams to the reservoir; the trend of thought in this connection seems to favor use of debris dams. But another method of holding debris out of the reservoir merits more attention than it has received. Normal upvalley aggradational processes may be accelerated by use of groins and other temporary and inexpensive structures so placed as to cause the stream to decrease its transporting efficiency (perhaps it would be better to say—so placed as to accentuate the natural tendency of the meandering or braided channel to be exceedingly inefficient). This method conforms with the well-recognized principle that, in dealing with rivers, better results may be achieved with less human effort by working with the water, rather than against it; river training to induce deposition is to impounding debris by damming as river training to maintain a navigable channel is to dredging. Training measures intelligently planned to increase the rate of upbuilding of the channel by deposition of bed load, and to increase the rate of vertical accretion on the floodplain by overbank deposition of suspended load, combined with suitable types of agricultural utilization of the valley floor as upbuilding proceeds, may provide a partial answer to the problem of reservoir silting. Quantitative data are needed to evaluate the practicability of controlled upvalley aggradation as alternative or supplementary to the debris dam system.

These suggestions apply to the simple case (as that of Lake Mead on the Colorado) where protection of the reservoir is the primary concern. The situation is quite different on the Rio Grande, where aggradation above the Elephant Butte reservoir will destroy valuable property on the valley floor. The engineer charged with corrective measures on the Rio Grande will face an interesting dilemma. He can, by increasing the efficiency of the channel, permit the stream to transport its load on a lower slope, thus delaying destruction of upvalley property and shortening the useful life of the reservoir, or he can, by decreasing the efficiency of the channel, force the river to develop a higher slope by aggradation, thus hastening destruction of upvalley property and lengthening the life of the reservoir.

LOWERING OF BASE LEVEL

Lowering in base level, is, insofar as the response of the stream is concerned, essentially the same as the lowering of a barrier in its path. Each small lowering of the control point steepens the gradient immediately upstream. Accelerated velocity in the steepened portion results in downcutting, and the steepening is propagated upvalley. Downcutting must continue until the slope is again completely adjusted to supply just the velocity required to transport all of the debris shed into the stream; as in the last case, and with the same qualifications, the final readjusted profile will tend to parallel the original profile.

A man-made change which corresponds with a lowering in base level is the local shortening of a stream by the elimination of meander loops; the general trend of the upvalley effects is indicated or, better, merely suggested by changes in the profile of the Mississippi River brought about by a series of artificial cutoffs and other channel improvements between 1929 and 1939. A generalized and more or less diagrammatic profile (Ferguson, 1939, p. 829) shows the new slope between the cutoffs essentially the same as the original slope, the changes in level resulting from the individual cutoffs being cumulative in an upvalley direction. At Arkansas City, at the head of the cutoffs, the river level was lowered about 15 feet. The effect was noted in 1939 at a gauge 107 miles above the head of the cutoffs, where there was a lowering of 2 or 3 feet in flood stage. The river has certainly not yet adjusted itself to the new conditions; the chief significance of the recorded profile changes to date is the sensitiveness of the stream to "lowering of base level", and the extremely rapid headward progression of the first effects of that lowering.

Macar, on the basis of studies of natural cutoffs in European and American rivers, concludes that the "steepened part" caused by a given cutoff moves headward in large streams several hundred times faster than the rate of downcutting of the stream; his profiles show subparallelism between the original and readjusted slopes (1934).

CLASSIFICATION OF CHANGES IN CONTROL

Changes of the type discussed above may be usefully divided into two catagories: (1) "upvalley", and (2) "downvalley" changes in controls; the terms are used, of course, with reference to a particular segment under consideration. As indicated in an earlier section the slope of the graded profile is determined primarily by load and discharge, which are "upvalley" factors; change in either or both of these factors calls for changes in *slope.* The level at which the profile is developed is determined by base level, which is a "downvalley" factor; change in base level calls for a change in the level of the profile but does not, directly, call for change in its slope. A single change usually affects the stream differently above and below the point where the change occurs; for example, if the load delivered by a tributary is greatly increased the effect on the trunk stream below the junction is increase in load, and above the junction rise in base level. The distinction between upvalley and downvalley changes in control is of special importance in interpreting the significance of parallel and converging terrace and valley-floor profiles.

Crustal movements, as tilting or warping of any segment of a graded stream, constitute a third category of changes. Such changes are neither upvalley nor downvalley, and they do not necessarily involve changes in load, discharge, or base level; they simply cause a forcible distortion of all earlier developed terrace and valley-floor profiles within the segment affected. The stream responds by building or downcutting or both. Except insofar as base level or factors which control slope are changed, either in consequence of the crustal movements directly or by reason of the upbuilding or downcutting of the stream, the completely readjusted profile will be developed at the same level and will have the same slope as the original profile.

REGRADING WITH PROGRESS OF THE EROSION CYCLE

A stream may lengthen during its life history by headward erosion, delta growth, and the development of meanders. Lengthening by the slow extension of ungraded headwaters and by capture involves additions of discharge and load to the stream. The contrasted effects of these additions have been treated earlier; their net effect on the graded portion of the stream is determined by their relative importance. The change, with the progress of the erosion cycle, from the relatively straight course of youth to the meandering course of maturity involves a systematic lengthening of the stream. This lengthening, together with that resulting from delta building, tends to decrease declivity and hence calls for aggradation to maintain the slope required for the transport of load. Whether aggradation will actually occur and how far upvalley its effects may be felt at any stage depends upon the relative rate of an accompanying opposed upvalley change, namely, the tendency for slow lowering of graded declivity resulting from decrease in load with advancement of the cycle. Green has developed an interesting mathematical treatment of the change in the form of the profile by which, with certain initial assumptions made, the past and future shifting of the zones of aggradation and degradation can be determined (Green, 1936; *see also* Miller, 1939; Lewis, 1945).

These changes may be difficult or impossible to detect in large, low-gradient streams of humid regions, especially because they tend to be masked by or confused with the effects of subsidence in the deltaic area and eustatic changes in sea level. Their effects are more clearly seen in high-declivity graded streams flowing from semiarid ranges to closed desert basins under such conditions that base level is raised *pari passu* with decrease in the declivity of the stream that accompanies reduction of the range. As shown by Johnson and others, the stream maintains its graded condition under these conditions by *regrading*, being engaged in aggradation in the lower levels at the same time that it is degrading on the rock-floored pediment, the line of demarcation between the contrasted zones shifting rangeward or basinward depending on the relative importance of the downvalley and upvalley factors (Johnson, 1932).

There is perhaps nothing seriously wrong with the statement that the stream maintains a graded condition while continuously lowering its slope during the process of the cycle of erosion; the statement is merely an oversimplification that is confusing because it is inconsistent. In a textbook discussion of the *cycle of erosion* it would seem preferable to state that, having developed an equilibrium slope under existing conditions at any given stage of the cycle, that is, having attained a condition of grade, the stream must continually alter its slope and its graded condition as controlling conditions change. The rate of change in controlling conditions is exceedingly slow, and the stream's adjustment to the controlling conditions is exceedingly delicate. The lag between change in conditions and adjustment is therefore so completely negligible that, viewed at any one time, the stream may be properly regarded as being graded. But if attention is focused on the orderly change in landforms during the erosion cycle, that is, *if the topic under discussion is the cycle*, then the emphasis should be not on the "static" equilibrium at any one time, but on the gradually shifting equilibrium over a long period of geologic time. In the course of the erosion cycle the stream maintains itself, not in any one graded condition, but in an infinite number of different graded states, each differing slightly from the last and each appropriate to the existing conditions.

If there is anything paradoxical or confusing in this relationship it escapes the writer. But even the best student may be hopelessly confused if the concept of grade and the wholly different concept of the erosion cycle are churned up together and administered in one dose. Grade must be explained in terms of channel processes and thoroughly understood as a condition of equilibrium in the stream as a transporting agent before the fourth dimension, geologic time, is introduced in a consideration of the role of the stream in the cycle.

An understanding of grade is essential for any theoretical analysis of the erosion cycle, but the principle geologic application of the concept is in interpretation of erosional and depositional landforms produced directly by the work of streams. The field worker in most continental areas finds that he must deal with streams that have at different times in the recent past engaged in episodes of downcutting and valley filling, both proceeding at varying rates and punctuated by pauses. The complex history of most modern stream valleys is undoubtedly due largely to the climatic fluctuations, crustal movements, and eustatic changes in sea level that characterize the Pleistocene. Valley floors and terrace remnants of several types and with various slopes in the same valley record the automatic tendency of the streams to adjust themselves to these changing conditions; even if equilibrium were never attained the theory of grade would be indispensable for any understanding of these strivings toward it. But, as shown by examples cited earlier, which could be multiplied a hundred fold, essential equilibrium is in fact often attained and maintained long enough for the production of distinctive landforms by long-continued lateral planation at the same level.

SHORT-TERM CHANGES

After the meandering habit is fully developed in any segment the length of the stream is not significantly altered by continued shifting of the meanders; the shortening effect of occasional cutoffs is cumulative only in the pages of Mark Twain's "Life on the Mississippi". That these local shortenings are compensated, over a period of years, by continued slow growth of all of the loops is indicated by the fact that, although 20 cutoffs between 1722 and 1884 shortened the Mississippi by about 249 miles, the river was by 1929 about the same as the original length (Pickels, 1941, p. 339).

Disturbance of equilibrium relations in the stream by sudden local shortenings and slow general lengthening are not inconsistent with the concept of grade as defined here. If a cutoff results in a "significant" break in the longitudinal profile the stream may be properly considered to be ungraded at that point; the height of the break that is "significant" depends simply on the point of view of the investigator. Schoklitsch's analysis of profile readjustments in the vicinity of a single cutoff, too long and not sufficiently pertinent to be summarized here, is an excellent illustration of readjustment on a small scale (1937, p. 153, 154; *see also* Shulits, 1936). The effect of a number of cutoffs was cited above, also from the engineering literature, to illustrate the very rapid headward propagation of a local steepening in the longitudinal profile. But in most cutoffs in streams flowing in alluvium the resulting steepening in the bed is less than its normal relief from bend to crossing, the break in the slope of the water surface is less than the difference between high and low water levels, and the increased velocity in the steepened part is less than the seasonal variation in velocity. Because breaks of this type usually leave no recognizable record in the valley-floor features, they may in many types of geologic studies be properly neglected in considering whether the stream is graded. With other short-term fluctuations in velocity, discharge, and load, they are considered to be covered by the expression "over a period of years" which is an essential part of the definition of grade.

DEPOSITS OF GRADED AND AGGRADING STREAMS

The body of detritus that floors the valley of a graded stream usually consists of three types of material, unlike in origin: (1) sand and gravel originally carried as bed load and deposited in the shifting channel, (2) an overlying sheet of sand and silt deposited from suspension in slow moving or ponded overbank waters, and (3) detritus not directly related to the work of the main stream, as talus and slope wash from the valley sides, and loess and wind-blown sand.[8] The maximum potential thickness of the channel gravel sheet equals the maximum potential depth of scour in the shifting channel during high-water stages; a thickness of a few tens of feet does not necessarily indicate that the stream has aggraded or is aggrading. The maximum thickness attainable by the overbank silt is determined by the height to which it can be built by successive overspreadings of the valley floor by flood waters without long-term change in the river level; the natural levee, as such, is not an evidence of aggradation. There is no well-defined limiting thickness for deposits of type (3); for example, over 150 feet of fan wash accumulated over the main stream channel gravel sheet in the Shoshone Valley in Wyoming during a period when the Shoshone River was not aggrading (Mackin, 1937, Pl. 1, p. 827–833).

True aggradation, which involves a systematic long-term rise in the river level, may be effected by a thickening of deposits of types (1), (2), or (3), or any combination of them.

An aggrading stream may produce a thick fill that consists largely or wholly of channel deposits. Numerous examples occur in valleys marginal to glaciated ranges, where the fill was deposited by proglacial streams and subsequently trenched so that its internal structure can be seen; exposures in the frontal scarp of a great fill terrace in the Cle Elum Valley in Washington, for instance, show over 200 feet of uniformly bedded gravels. This preponderance of channel deposits certainly does not mean that the Pleistocene Cle Elum River carried no suspended load, nor does it mean that the successively higher valley floors formed during the period of aggradation carried no veneers of overbank silt.

Aggradation may, on the other hand, produce a fill made up largely of silt and lacustrine clay—the Oligocene White River deposits of the Great Plains are a case in point. The prevailing fine texture of the White River beds certainly does not represent the loads carried by the Oligocene streams; lenses and stringers of channel gravel, contrasting sharply with the overbank silts and clays, occur throughout the White River sediments (Osborn, 1929, p. 103–109).

The thick aggradational fill in the Kickapoo Valley in Wisconsin consists chiefly of slope wash derived from the local valley sides (Bates, 1939, p. 870–876). This is perhaps an exceptional case, but greater or less amounts of local wash, partly re-

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[8] For a more elaborate classification of valley floor materials *see* Happ, Rittenhouse and Dobsen, 1940, p. 22–31.

worked and interfingering with main stream deposits, are to be expected in all valley fills.

Contrasts between aggradational fills of different types depend largely on contrasts in the rate of lateral shifting of the depositing stream relative to the rate of upbuilding of the deposit and, especially, on the mechanism of lateral shifting, whether by meander swing and sweep processes (Melton, 1936) or by avulsion as in the braided stream. These habits depend in turn on slope, discharge, load, channel characteristics, and the hydrographic regimen of the stream, resistance of the valley-floor materials to lateral cutting, vegetative cover on the valley-floor, distance from source of the detritus and from the mouth of the stream, relief and erosional processes on adjacent uplands, and other factors that cannot be evaluated here. The points to be made are simply: (1) that *deposits* formed by or associated with aggrading streams differ markedly from the *loads* carried by them, (2) that distinguishing between channel and overbank deposits is the first essential step in interpreting modern valley fills or ancient fluviatile sediments, and (3) that, even after this distinction is made it is virtually impossible to work directly from the grade sizes represented in the channel deposits to the characteristics of the depositing streams because there is no simple relationship between the deposits of an aggrading stream and such partly interdependent factors as slope, discharge, channel characteristics, velocity, and load. We cannot proceed directly from laboratory-determined laws relating to stream *transportation processes* to interpretation of ancient stream *deposits*. An alternative and promising route of attack on the problem is via study of deposits now being formed by natural streams of many types to determine whether the sum total of all of the characteristics of given deposits is uniquely related to the particular modern streams by which they are being formed, and to proceed thence to an understanding of the characteristics of ancient streams by comparison of their deposits with deposits of modern streams of known characteristics.

Here again, as in every phase of the study of streams, there is a broad field in which the interests of the geologist and the engineer overlap. For example, correct evaluation of the factors that control what proportion of overbank silts and channel gravels are incorporated in an aggrading valley fill under natural conditions may permit intelligent modification of these factors to alter, to the advantage of a reservoir, the ratio of bed load to suspended load in the debris charge delivered to the reservoir by the aggrading streams that enter it. The marked decrease in the rate of storage depletion in Lake Mac Millan brought about by accidental introduction and development of a dense growth of saltcedar (tamarisk) on the aggradational flood plain above the reservoir (Eakin and Brown, 1939, p. 17–18; Walter, in discussion of Taylor, 1929, p. 1722–1725) illustrates the effectiveness of modification of only one of the factors controlling silt deposition on the valley floor.

SECONDARY EFFECTS OF AGGRADATION

DECREASE IN SUPPLIED LOAD

Aggradation (for whatever reason) involves burial of the lower waste-shedding slopes of the valley sides, and a consequent reduction in the load supplied to the stream system as a whole. Usually far more important, back-filling in valleys tributary to the aggrading trunk stream tends to reduce the load delivered by the tributaries. In the extreme case, aggradation by the main stream may so far outstrip upbuilding by its tributaries that the lower portions of their valleys are ponded; settling basins formed in this manner serve to entrap all or most of the clastic waste carried by the tributaries. (For examples, *see* Lobeck, 1939.) In general, decrease in load calls for decrease in the slope of the main stream.

DECREASE IN DISCHARGE

A stream flowing in a valley cut in rock normally carries most of the runoff from its drainage basin as surface discharge. But in an aggraded valley a considerable part of the runoff may pass through the detrital filling as underflow, with a consequent reduction in surface discharge available for the transportation of load. In general, decrease in discharge calls for an increase in declivity.

CHANGE IN CHANNEL CHARACTERISTICS

Aggradation often involves a shoaling and widening of the channel, and it may cause very marked changes in the stream's characteristics, as from a meandering to a braided habit. These changes usually tend to reduce the efficiency of the channel and therefore to require an increase in declivity.

GENERAL STATEMENT

Changes of the types listed above are more or less incidental results of the reaction (aggradation) of a stream to a given "primary" change in control. Since they, in turn, call for modifications in declivity, they may be regarded as "secondary" controlling factors. They operate especially during the period of aggradation, but they may continue to affect the stream long after it has readjusted itself to the new conditions. Thus, while certain "downvalley" changes in control (as rise of base level) do not directly call for change in slope, the eventual readjusted profile may, under different circumstances, be steeper or less steep than the original profile because of secondary chain-reaction effects. Any attempt to predict the final form or slope of the readjusted profile by evaluating the effects of a given man-made primary change in controls which fails to take into account these secondary changes in slope-controlling factors is liable to serious error. In general, the net effect on declivity of any primary change in control will be the algebraic sum of the parallel or opposed effects of the primary change and the associated secondary changes.

PROFILES OF ADJUSTED AND DISADJUSTED STREAMS

SORTING IN THE GRADED STREAM

As the velocity of the graded stream increases with seasonal increase in discharge, progressively coarser grains are set in motion up to the limits of the competence of the current and the supply of detritus available in the bed and banks. As velocity decreases with passing of the high-water stage, the materials in motion are thrown down in order of decreasing grain size. Every part of the channel deposit that veneers the valley floor is worked over again and again as the stream shifts laterally, each time with selective deposition of the coarser materials and a winnowing out of the fines. For this reason the valley-floor channel deposits in any part of the valley contain notably higher proportions of coarse materials than the bed load in transit through that segment of the valley in any representative period of time. There is, in this sense, a sorting process in operation in the graded stream.

Because the finer grains in the bed load are set in motion earlier, move faster while in motion, and are retained in motion longer during each seasonal fluctuation in discharge, there is a "running ahead of the fines" in the channel of the graded stream. But this "running ahead of the fines" does not cause a downvalley decrease in the caliber of the load handled by the stream. The fines move faster toward and into any given segment, but they also move faster through and away from that segment; the average grain size in the bed load moving through any segment is the same as though all the grain sizes moved at the same rate. The primary cause of the downvalley decrease in caliber of load in the graded stream is attrition, not sorting.

The same conclusion holds for the channel gravel sheet which veneers the valley floor. Every boulder or pebble that is lodged in the valley-floor alluvium in any segment of the valley at any period of time is likely to be moved and lodged again farther downvalley in successive intervals of time. Downvalley decrease in the grain sizes making up the channel gravel sheet is due, not to sorting, but to attrition during stages of movement in the channel, possibly accentuated by weathering during periods of lodgement.

SORTING IN THE AGGRADING STREAM

It has been noted earlier that aggradation may be effected either by a downvalley movement of a wave or waves of steepened declivity, or by an upvalley propagation of a wave or waves of flattened declivity. The essential relationship in both cases is that of a given segment shedding debris into an adjacent (downvalley) segment that is not quite adjusted to the transportation of that debris. Under these circumstances the operation of the seasonally expanding and contracting channel combines with a deficiency in velocity to assure that the coarsest fractions of the load entering any disadjusted segment will be deposited in that segment. Since the channel of the stream is raised as aggradation proceeds, the materials so deposited will not be subject to reworking and continued downvalley movement. There is in the aggrading stream, in other words, a "permanent withdrawal from circulation" of the coarser fractions of the bed load, and the "running ahead of the fines" in this case makes for a real and substantial downvalley decrease in the caliber of the load in transit, and in the materials deposited by the stream as it aggrades.

EXCHANGE IN GRADED AND AGGRADING STREAMS

The channel of the graded stream, shifting back and forth in a valley-floor gravel sheet made up largely of channel deposits which may be somewhat coarser than the average bed load in transit, has little chance to decrease the caliber of its load by the process of exchange. But the channel of the aggrading stream may shift laterally in alluvium consisting essentially of contemporaneous overbank deposits. In this case the materials added to the load of the stream by scour and caving at the outside of a shifting bend may be largely fine silts and sands, but the coarser fractions of the bed load supplied from upvalley tend to be deposited and left behind as the channel shifts. The term exchange as used here does not imply any equivalence in weight or bulk of the material set in motion and deposited; the stream simply cuts at the outside of its bends because inertia normally holds the strongest current against the outside bank; it deposits the coarsest fractions of the load in transit on its channel floor with every seasonal fluctuation and leaves some of these deposits behind as the channel shifts. In the aggrading stream, in contrast to the graded stream, this type of exchange may result in a notable downvalley decrease in the caliber of load in transit.

EFFECT OF DOWNVALLEY CHANGE IN CALIBER OF LOAD ON THE FORM OF THE PROFILE

It has been repeatedly emphasized that the declivity of the graded stream is controlled by load (and other factors); the declivity is adjusted to furnish just the velocity required for the transportation of all the load supplied to the stream. In the aggrading stream the supplied load does not control declivity in the same degree because, by definition, *all* of the supplied load is not transported. But it is important to recognize that the amount of material moved through any segment of the channel of the aggrading stream in any interval of time is enormously greater than the amount deposited, and that even in the aggrading stream the declivity of each segment is *approximately* adjusted to the load in transit through that segment. In general, with decrease in the discrepancy between the supplied load and the load in transit, the aggrading stream approaches the graded condition.

No generalization can be made with regard to the average steepness of the profiles of graded and aggrading streams as such; both may vary from a small fraction of a foot to hundreds of feet per mile. But an exceedingly useful generalization can be made with regard to a contrast in the *form* of the profiles of graded and aggrading streams. Since declivity is in general adjusted to caliber of load in transit, and since the downvalley decrease in caliber of load in aggrading streams (by attrition, sorting, and exchange) is much more rapid than the downvalley decrease in caliber of load in graded streams (by attrition), it follows that declivity should decrease in a downvalley direction much more rapidly in the aggrading stream than in the graded stream under otherwise similar conditions. The profile of aggradation should be, in other words, more strongly "concave upward" than the graded profile. Thus, while the profile of the graded stream usually shows no tendency to be asymptotic with respect to a horizontal plane passing through a downvalley control point, the profile of the aggrading stream should and usually does show a definite tendency in this direction.

The writer has found two "rules" that follow from the discussion above to be useful tools in field study and interpretation of terraces of many types in stream valleys: (1) If there is any considerable length of stream upvalley from a given segment, aggradational channel deposits in that segment are so consistently finer in grain size than earlier or later deposits formed when the stream was at grade that variation in grain size and sorting serves as a criterion, for example, in distinguishing between channel deposits laid down in a valley-filling stage and the channel gravel sheet that mantles terraces cut in the fill during a subsequent degradational stage. (2) Aggradational profiles (recorded by terrace remnants) are usually steeper than earlier or later graded profiles in the upper parts of proglacial valleys, but the contrast in slope decreases in a downvalley direction and may be reversed, so that the aggradational profile is less steep than the graded profile in the vicinity of a downvalley control point.

CONCLUSIONS

> "Let us suppose that a stream endowed with a constant volume of water, is at some point continuously supplied with as great a load as it is capable of carrying. For so great a distance as its velocity remains the same, it will neither corrade (downward) nor deposit, but will leave the slope of its bed unchanged. But if in its progress it reaches a place where a less declivity of bed gives a diminished velocity, its capacity for transportation will become less than the load and part of the load will be deposited. Or if in its progress it reaches a place where a greater declivity of bed gives an increased velocity, the capacity for transportation will become greater than the load and there will be corrasion of the bed. In this way a stream which has a supply of debris equal to its capacity, tends to build up the gentler slopes of its bed and cut away the steeper. It tends to establish a single uniform slope. . . ."
>
> "Every segment is a member of a series, receiving the water and waste of the segment above it, and discharging its own water and waste upon the segment below. If one member of the series is eroded with exceptional rapidity, two things immediately result; first, the member above has its level of discharge lowered, and its rate of erosion is thereby increased; and second, the member below, being clogged with an exceptional load of detritus, has its rate of erosion diminished. The acceleration above and the retardation below, diminish the declivity of the member in which the disturbance originated; and as the declivity is reduced the rate of erosion is likewise reduced.
>
> "But the effect does not stop here. The disturbance which has been transferred from one member of the series to the two which adjoin it, is by them transmitted to others, and does not cease until it has reached the confines of the drainage basin. For in each basin all lines of drainage unite in a main line, and a disturbance on any line is communicated through it to the main line and thence to every tributary. And as any member of the system may influence all the others, so each member is influenced by every other. There is interdependence throughout the system."

These paragraphs, taken with minor changes in wording from the Henry Mountain report, dated 1877, set forth the essence of Gilbert's idea of grade in streams; additions and modifications discussed in the present paper are chiefly matters of qualifying detail. The principal conclusion, that the concept of the graded stream as a system in equilibrium is valid, is based on:

(1) Citation of broad valley floors cut by long-continued planation at the same level by high-gradient streams crossing rock types of varying resistance to corrasion;

(2) Analysis of the form of the longitudinal profile developed and maintained by the graded stream under stable conditions, demonstrating by citation of cases that, in each segment, slope is adjusted to provide, with available discharge and under prevailing channel conditions, just the velocity required for transportation of all of the load supplied to that segment without regard for variation in resistance to corrasion in the subjacent materials; and,

(3) an outline of the manner in which graded streams readjust themselves to natural and man-made changes in controlling conditions of several types, demonstrating that the stream responds to such changes always so as to "absorb the effect of the stress", and thus exhibits the chief and diagnostic characteristic of the equilibrium system.

A critical point in connection with (2) is that, because in a trunk stream conditions controlling slope do not vary systematically from segment to segment, the longitudinal profile cannot be a simple mathematical curve. This conclusion is qualitative; if, in conformity with it, we cease to smooth out real departures from uniformity and center the attack on them, with an adequate understanding of the genetic relationships of the independent and interdependent factors involved, then mathematical analysis of longitudinal profiles will advance our knowledge of streams.

A second generalization, important because it has been so generally neglected in geologic writings, is that the slope of the graded profile is adjusted to, or controlled by, not only the classic "load and discharge" but also the cross-sectional form and alignment of the channel—the more efficient the channel, the lower the slope.

In connection with (3) the present study tends to confirm the standard geologic view that streams readjust themselves to new conditions primarily by adjustments in slope, and only in minor degree by modification of the channel section. This statement is so phrased as to avoid any semblance of a "law"—certainly no fetish attaches to slope, and each individual case must be judged on the basis of the evidence. But it does appear that, confronted by changed conditions that call for increased or decreased energy for transportation, the stream usually responds by increasing or decreasing its total energy by appropriate adjustments in slope rather than by effecting economies in the energy dissipated in friction.

Additional generalizations include the distinction between "upvalley" and "downvalley" changes in control and between "upvalley" and "downvalley" reactions of the stream to a given change, the contrast between the form of the disadjusted profile during the period of readjustment and the final readjusted profile, and the effect of secondary changes in control on the slope of the readjusted profile.

With a few minor lapses, this paper does not treat the practical implications of the concept of grade. In geology these ramify widely, ranging from the power of rivers to corrade laterally to interpretation of ancient fluvial sediments and the origin of unconformities beneath and within them. In connection with control of rivers by men, a safe general implication is that the engineer who alters natural equilibrium relations by diversion or damming or channel-improvement measures will often find that he has a bull by the tail and is unable to let go—as he continues to correct or suppress undesirable phases of the chain reaction of the stream to the initial "stress" he will necessarily place increasing emphasis on study of the genetic aspects of the equilibrium in order that he may work *with* rivers, rather than merely *on* them. It is certain that the long-term response of streams to the operations of the present generation of engineers will provide much employment for future generations of engineers and lawyers.

In this connection the most important point brought out by the study may well be the striking analogy between the streams' response to the works of man and to accidents and interruptions due to geologic causes. Nature has brought to bear on streams nearly all of the changes in controlling conditions that are involved in modern engineering works; the record of the long-term reaction of rivers to past geologic changes that is revealed by terraces and in dissected valley fills should contribute much to an understanding of the future of streams that man seeks to control, and will call for changes in design. Conversely, every advance in knowledge of erosional, transportation, and sedimentation processes deriving from engineering investigations will increase the geologist's ability to interpret the record of the past. As the engineer becomes more and more concerned with the genetic aspects of his especial problems (as he must), and as the geologist learns more about the quantitative aspects of his especial problems (as *he* must), it will become evident that the problems are in large measure the same. As Rubey (1931b) puts it, there is "a need for close cooperation among *students of stream-work.*"

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MANUSCRIPT RECEIVED BY THE SECRETARY OF THE SOCIETY, JUNE 3, 1946.