DRAINAGE DENSITY AND STREAMFLOW

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ABSTRACT

Drainage density, surface-water discharge, and ground-water movement are shown to be parts of a single physical system. A mathematical model for this system, which was developed by C. E. Jacob, can be expressed in the equation $T = WD^{-2}/8h_0$ in which T is transmissibility, W is recharge, D is drainage density, and h_0 is the height of the water table at the water table divide. The rate of ground-water discharge into streams, or base flow (Q_b), is dependent on and varies directly with transmissibility of the terrane. If W and h_0 are constant, $Q_b \propto D^{-2}$. Thirteen small, almost monolithologic basins, located in a climatic area in the eastern U.S. where recharge is nearly a constant, were found to have a relation of base-flow discharge to drainage density in the form of Q_b per mi$^2 = 14D^{-2}$. The field relations, therefore, agree with that predicted by the Jacob model.

If transmissibility controls the relative amount of precipitation which enters the ground as contrasted with that which flows off the surface, surface-water or overland discharge should vary inversely with transmissibility. It was found that flood runoff as measured by the mean annual flood ($Q_{2.33}$) varies with drainage density in the form of $Q_{2.33}$ per mi$^2 = 1.3D^2$. The close relation of mean annual flood to drainage density in 15 basins was not affected by large differences among the basins in relief, valley-side slope, stream slope, or amount and intensity of precipitation. It is concluded that drainage density is adjusted to the most efficient removal of flood runoff and that the mean annual flood intensity is due predominantly to terrane transmissibility.

INTRODUCTION

This report describes the results of an investigation of some of the relations between hydrology and geomorphology. The hydrology of a stream basin involves (1) the overland runoff from the basin, (2) the ground-water recharge rate, which is dependent upon the amount of precipitation in excess of evapotranspiration losses and upon the infiltration capacity of the soil mantle, and (3) the permeability of the bedrock, which affects the rate of yield of ground water to wells, to springs, and to the streams which drain the basin. Stream-flow characteristics were examined in terms of base flow and of height of flood peaks.

The geomorphic character of drainage basins, following techniques first suggested by Horton (1945), may be measured or described in a variety of ways. For example, Strahler (1958, p. 282–283) has listed 37 such properties or parameters. In a general way these properties may be divided into four classes: (1) length or geometric properties of the drainage net, (2) shape or area of the drainage basin, (3) relation of the drainage net to the basin area, and (4) relief aspects of the basin. It has been assumed by hydrologists and geomorphologists that certain relations must exist between runoff characteristics and topographic or geomorphic characteristics of stream basins. Much effort has been devoted to statistical correlation of these relations without a clear understanding of how runoff is related to the geomorphic and geologic character of the basins. This paper advances the theory that runoff and drainage density are genetically and predictably related to the transmissibility of the underlying rock terranes.

The purpose of this research was not to develop a means of predicting stream-flow characteristics from a landform characteristic. It was to attempt to solve a complex geohydrologic problem by defining a physical system in terms of: (1) the elements of the system, (2) how it operates, and (3) why it operates in the observed fashion. The theory resulting from this type of analysis, if valid, illustrates the basic simplicity and rationality of natural processes.

During the progress of research, much effort was devoted to statistical analysis of the data. While very good correlation coefficients (0.96 to 0.98) were obtained the procedure shed no light on the physical cause and effects of the processes. The present theory, formed by inductive and deductive reasoning, is so basically simple that the correlation can be shown adequately in simple graph form and equations can be solved directly by graphical means.

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PREVIOUS STUDIES

Early studies of the relation of topographic character of drainage basins to basin-runoff characteristics have been described by Langbein (1947, p. 128–129). The topographic parameters considered important in these studies were: area, channel slope, stream pattern, average basin width, mean length of travel (channel length), and mean relief measured from gaging station.

In 1939 and 1940, Langbein (1947) and his coworkers, through assistance from the Works Project Administration of the Federal Works Agency, compiled a large variety of topographic measurements from 340 drainage basins in the northeastern United States. The basins ranged in area from 1.6 to 7,797 square miles. The parameters determined were drainage area, length of streams, drainage density, land slope, channel slope, area-altitude distribution, and area of water bodies. No attempt was made to correlate these topographic measurements with streamflow properties. The measurements were compiled for future studies which would determine their effects on the runoff of streams.

Such studies have been carried on in the Water Resources Division of the U.S. Geological Survey by Benson (1959), who participated in the studies reported by Langbein in 1947. As a result of graphical correlation of several topographic parameters with flood flow for 90 New England drainage basins, Benson determined that basin area and channel slope were the most effective determinants of flood flow. These parameters were then statistically computed in correlation with records of 170 gaging stations.

Benson's studies were made entirely of streams in New England where the bedrock lithology, with the exception of the Triassic rocks of the Connecticut

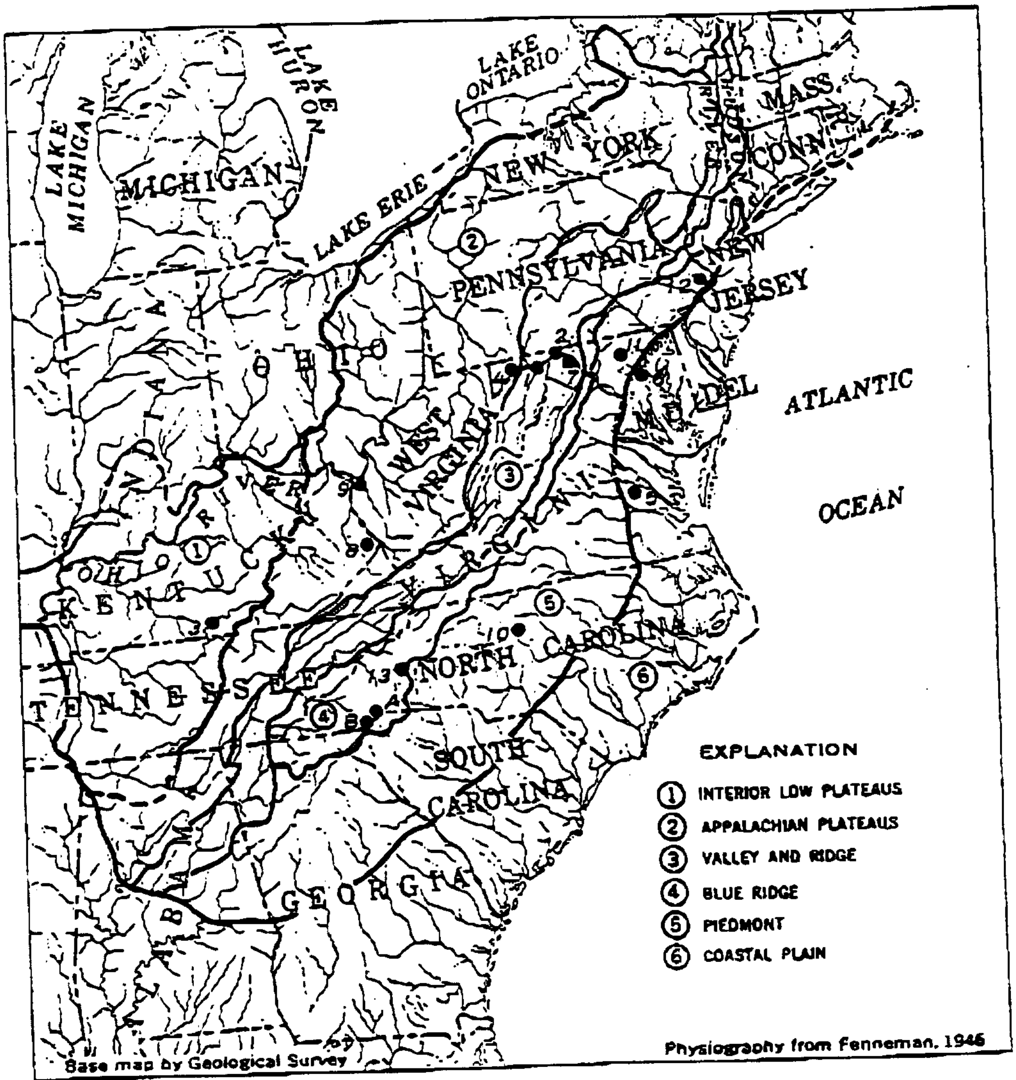

FIGURE 1.—Map showing location of drainage basins studied and discussed in this report. Uncircled numbers and letters correspond to those in table and on graphs.

basin and the Carboniferous rocks of the southeastern New England region, is largely a hydrologically homogeneous complex of igneous and metamorphic rocks. Glacial erosion has modified the land surface of the crystalline rocks, and glacial deposits mantle much of the area. Drainage density among the basins studied ranged from 1.1 to 2.35, as measured on 1:62,500 scale maps by Langbein.

METHODS OF STUDY

The area of study comprises the central and eastern United States south of the Wisconsin glacial border. This area was chosen because the landscape is the product of normal denudational processes; landscapes shaped or modified by glacial erosion or deposition, such as those studied by Benson (1959) were thus excluded. The area is generally homogeneous in climate. Average annual precipitation ranges from about 40 to 50 inches and average annual temperature ranges from 50° to 60° F. Four criteria were used in selecting basins having a variety of bedrock geology: (1) homogeneity of bedrock, (2) availability of 1:24,000-scale topographic maps, (3) availability of discharge records, (4) lack of appreciable flow regulation of the streams. These limitations resulted in an unavoidably small sample. The basins, their geology, and their hydrologic and geomorphic parameters are listed in the following table, and their locations are shown in figure 1.

Drainage densities were obtained by sampling randomly distributed 1-square-mile plots, whose total area is generally not less than 25 percent of the total basin area. Drainage density is a measurement of the sum of the channel lengths per unit area. It is generally expressed in terms of miles of channel per square mile. Horton (1945, pp. 283–284) and Langbein (1947, p. 133) determined drainage density by measurement of the blue streamlines on topographic maps having a scale of 1:62,500. Langbein (p. 133) pointed out that the number of small headwater streams shown on these maps would vary with the season and wetness of the year in which the survey was made, as well as with the judgment of the topographer and cartographer.

The current practice, and the method used in this study, is to show drainage lines wherever a cusp or V-notch of a contour line indicates a channel. By this method, the drainage densities of the basins, on topographic maps having a scale of 1:24,000, range from 3.0 to 9.5. The measurements are believed to be of consistent accuracy because the maps used were all 1:24,000 scale maps and the measurements were made by one operator, the author. Delineation of drainage lines was generally conservative; therefore, drainage-density values, particularly in the higher values, may be lower than those ordinarily measured.

An accurate method of distinguishing the ground-water discharge component of streamflow from the surface- or overland-flow component has not yet been developed. Estimates of ground-water discharge of streams, or base flow, are usually made by assuming that all flood peaks are overland-flow discharge and

Basin		Bedrock lithology	Area (sq mi)	Drainage density	Average minimum monthly flow (cfs per sq mi)	Mean annual flood (cfs per sq mi)	Mean annual precipitation (inches)
Locality (fig. 1)	Gaging station						
1	Sawpit Run near Oldtown, Md.	Shale	5	7.6	0.13	62	38
2	Little Tonoloway Creek near Hancock, Md.	Shale and shale interbedded with sandstone.	16.9	7.0	.29	44	38
3	South Fork Little Barren River near Edmonton, Ky.	Limestone and cherty limestone	18.1	9.5	.26	77	49
4	Crabtree Creek near Swanton, Md.	Sandstone (60–80 percent) and some shale.	16.7	4.2	.61	32	42
5	Totopotomoy Creek near Atlee, Va.	Sand and gravel	6.0	4.7	.35	23	45
6	Sawmill Creek near Glen Burnie, Md.	Sand and clay	5.1	3.0	1.22	19	45
7	Tuscarora Creek near Martinsburg, W. Va.	Limestone	11.3	3.6	.80	14	40
8	John's Creek near Meta, Ky.	Sandstone, shale, limestone, and coal.	55.7	6.3	.27	44	44
9	Fourpole Creek near Huntington, W. Va.	Sandstone, shale, limestone, and coal.	20.9	8.0	.10	86	43
10	East Fork Deep River near High Point, N.C.	Schist, slate, and greenstone	14.7	8.0	.36	102	46
11	Piney Run near Sykesville, Md.	Granite	11.4	6.0	.84	75	45
12	Ridley Creek near Moyland, Pa.	Gabbro and schist	31.9	5.0	.83	38	47
13	Beetree Creek near Swananoa, N.C.	Gneiss	5.5	5.6	.91	44	49
A	Crab Creek near Penrose, N.C.	Granite	10.9	7.4	1.86	72	62
B	Catheys Creek near Brevard, N.C.	Gneiss	11.7	8.0	2.45	67	63

that the remaining lower flow segments of the hydrograph are base flow. Many surface-water hydrologists believe that the base flow of many streams is largely derived from bank storage, rather than from discharge from the ground water. On the other hand, there are large areas of highly permeable sand formations in the Atlantic Coastal Plain where there is never any discernible overland-flow runoff and where virtually all discharge, including flood peaks, is ground-water discharge. Because of this uncertainty in the definition and computation of base-flow or ground-water discharge, the author used as a measure of base flow average minimum monthly flow, computed by averaging 6 years of monthly minimum discharge measurements.

The mean annual flood is determined by plotting annual maximum-flood discharges on modified probability graph paper in which the discharge peaks are plotted against recurrence interval. Gumbel (1958, p. 177) has pointed out that the return or recurrence interval for the mean annual flood is 2.2328 years, the median annual flood is 2 years and the most probable annual flood is 1.582 years. The U.S. Geological Survey has adopted the mean annual flood ($Q_{2.33}$) as its standard reference value. According to Wolman and Miller (1960, p. 60) most of the work of stream erosion is done by frequent flows of moderate magnitude with a recurrence interval of 1 to 2 years. The use of the $Q_{2.33}$ flood in the present correlation study is, therefore, generally in scale with the magnitude and frequency of floods which Wolman and Miller have demonstrated to be most important in geomorphic processes.

THE JACOB WATER-TABLE MODEL

Models depicting the relation of the ground-water table to ground-water drainage have been made by Horton (1936) and Jacob (1943, 1944).

Horton (1936, p. 346) considered the shape of the water table as a parabola. He stated that the elevation of the water table (h) at any point X distance from the draining stream is:

$$h=\sqrt{h_0^2+2\left(\frac{\alpha}{K_i}\right)\left(L_o x-\frac{x^2}{2}\right)}. \qquad (1)$$

In this equation, h_0 is the elevation of the draining stream, K_i is the transmission capacity of the soil, L_o is the distance from the stream to the ground-water divide, and α is the rate of accretion of recharge to the aquifer, with α being equal to or less than the infiltration capacity.

In 1943 and 1944, Jacob developed a parabolic-type equation for ground-water flow in a homogeneous aquifer of large thickness and having uniform accretion from precipitation (1943, p. 566). The model for this equation is shown in figure 2. In a simplified form the equation states:

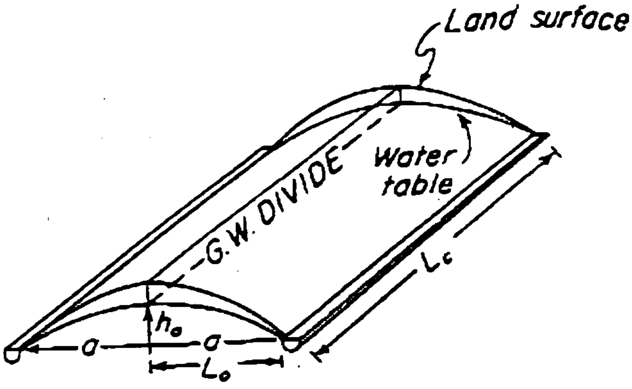

FIGURE 2.—Ground-water table base-flow model after Jacob (1943).

$$h_0=\frac{a^2W}{2T} \qquad (2)$$

where h_0 is the height of the water table above the draining stream, as measured at the ground-water divide, a is the distance from the water-table divide to the stream, W is the rate of accretion to the water table (recharge), and T is the transmissibility (the volume rate of flow through a vertical strip of the aquifer of unit width under a unit hydraulic gradient). Thus, according to equation 2, the height of the water table at the water-table divide is proportional to the square of the distance of the divide from the drainage stream and to the rate of ground-water recharge. The value h_0 is also inversely proportional to transmissibility.

APPLICATION OF THE JACOB MODEL TO LANDFORM AND STREAMFLOW CHARACTERISTICS

Jacob (1944, p. 938–939) computed the transmissibility of the Magothy sand aquifer of Long Island from a water-table contour map of the island and estimated recharge as about 60 percent of the average annual precipitation. He used the following equation for his computation:

$$T=\frac{a^2W}{2h_0}. \qquad (3)$$

The Jacob ground-water model can be related to a landform model by assuming that the land surface is also a parabola coincident with the water table. Then, the symbol a is equal to L_o or length of overland flow. Thus:

$$T=\frac{L_o^2W}{2h_0}. \qquad (4)$$

Horton (1945, p. 284) pointed out that the average length of overland flow (L_o) is, in most cases, approximately half the average distance between the stream channels and is therefore approximately equal to half the reciprocal of drainage density (D), or

$$L_0 = \frac{1}{2D}. \tag{5}$$

Substituting equation 5 in equation 4 the equation becomes:

$$T = \frac{W}{8D^2 h_0} \tag{6}$$

or

$$T = \frac{WD^{-2}}{8h_0}. \tag{7}$$

Jacob's water-table model as used on long Island, simulates an aquifer having parallel boundaries at a constant head (the sea level on both sides of the island). Jacob pointed out, however, that a serious weakness of the Long Island study was the impracticability of measuring the ground-water discharge along the northern and southern shores of the island. He suggested (1944, p. 939) that the model could be tested more effectively under field conditions where the parallel shores would be replaced by streams draining the water discharged from the water table aquifer. This model is shown in figure 2. The ground-water discharge into such streams would be dependent upon the transmissibility of the aquifer, and this discharge could be measured by the gain in flow per unit length of stream.

Inasmuch as ground-water discharge into streams would vary directly with the transmissibility, ground-water discharge or base flow (Q_b) should also vary according to equation 7 or:

$$Q_b \propto \frac{WD^{-2}}{8h_0} \tag{8}$$

W and h_0 remaining constant, base flow should be related to drainage density in the form:

$$Q_b \propto D^{-2} \tag{9}$$

It may be deduced that as transmissibility decreases, the amount or rate of movement of ground water passing through the system decreases and a proportionately greater percentage of the precipitation is forced to flow directly into the streams in flow over the land surface. Streamflow, therefore, would be derived from ground-water discharge plus overland flow. Both components of discharge should vary inversely in their relative contribution to stream discharge in a regular and predictable system controlled by the transmissibility of the water-table aquifer. As T increases, ground-water discharge into streams increases and surface discharge decreases. As T decreases, there would be a corresponding decrease in base flow and increase in surface discharge.

Equation 6, which was derived from Jacob's basic equation, states that transmissibility varies inversely with drainage density squared. Thus as transmissibility increases, drainage density would decrease, and as transmissibility decreases, drainage density would increase.

This relationship may be examined from two different viewpoints. Horton (1945, p. 320) has stated that erosion will not take place on a slope until the available eroding force exceeds the resistance of the surficial materials to erosion. This eroding force increases downslope from the watershed line to the point where the eroding force becomes equal to the resistance to erosion. He named this distance the "critical distance," and the belt of land surface within the critical distance was termed the "belt of no erosion." One of the most important factors in determining the width of the belt of no erosion is the infiltration capacity of the soil. Simply stated, the greater the infiltration capacity, the less will be the amount of surface runoff. As infiltration capacity increases, the critical distance also increases because a greater slope length is required to accumulate overland flow of sufficient depth and velocity to start erosion. Thus the infiltration capacity is chiefly responsible for determining the width of the belt of no erosion and the width of the spacing of streams or channels which carry away surface runoff.

The infiltration capacity of the soil may be regarded as one part of the general capacity of a terrane to receive infiltering precipitation and to transmit it by unsaturated flow through the vadose zone above the water table and by saturated flow through the aquifer to the streams draining the ground water. This capacity is here termed "terrane transmissibility." It is recognized that this extends the strict meaning of transmissibility, which is a measure only of saturated flow.

Another way of regarding the relation of transmissibility to drainage density is to consider that as T decreases, a concurrent progressive increase in surface flow will occur. Increase in proliferation of drainage channels would provide a more efficient means of transporting the water off the land, and as the water to be so removed increases (with decreasing T) the proliferation and closeness of spacing of the drainage channels would increase. The channel spacing or geometry should operate at peak efficiency during periods of flood runoff. Since flood runoff (Q_f) of streams varies in magnitude inversely with the magnitude of their base flow runoff (Q_b), ($Q_b \propto 1/Q_f$), and according to equation 9 $Q_b \propto D^{-2}$, then flood runoff should vary with the positive second power of drainage density, or

$$Qf \propto D^2. \qquad (10)$$

Chorley and Morgan (1962) have compared the morphometry of two areas of crystalline and metamorphic rocks: the Unaka Mountains of Tennessee and North Carolina, and Dartmoor, England. Morphometric characteristics which were measured included number of stream segments, mean stream lengths, mean basin areas, mean stream-channel slopes, mean relief of fourth-order basins, mean basin slopes, and mean drainage density. Relief and drainage density were the only parameters which showed significant differences between the two areas. The difference in drainage density between the two areas (Dartmoor, $D=3.4$; Unakas, $D=11.2$) was ascribed to differences in runoff intensity caused by more intense rainfall and greater mean basin slopes. They concluded that the channel system is geared to conditions of maximum runoff. These conclusions are only partially in agreement both with the theoretical and observed relations described in the present paper.

THE RELATION OF BASE FLOW TO RECHARGE AND DRAINAGE DENSITY

Figure 3*A* shows the relation of drainage density to base flow per square mile for 13 basins previously described. As previously stated, average annual precipitation in the 13 basin areas ranges from about 40 to 50 inches and average temperatures range from 50° to 60° F. Recharge (W) is dependent upon the amount of precipitation less the amount of evapotranspiration losses, which is dependent largely on temperature. The variations in rainfall and temperature among the 13 basins are within a sufficiently small range that recharge can be considered to be approximately a constant for the 13 basins. The relation of base flow per unit area to drainage density can be delineated by a regression line which has a slope and intercept such that:

$$Q_b \text{ per mi}^2 = 14D^{-2}$$

Equation 8 states that base-flow rate is also directly proportional to recharge. Two basins at the southern tip of the Blue Ridge province were examined to determine qualitatively the effect of significantly higher precipitation or recharge on base flow. In this area annual precipitation is more than 60 inches. The two basins are those of Crab Creek near Penrose, N.C. (*A*), and Catheys Creek near Brevard, N.C. (*B*). (See figs. 1, 3.) Their position on the diagram in figure 3*A* shows a much higher rate of base flow which may be due to the higher recharge rate. The gaging stations for both basins are, however, located in alluvium-filled valleys; therefore, the higher base flow discharges may be due in part to higher transmissibility of the alluvium.

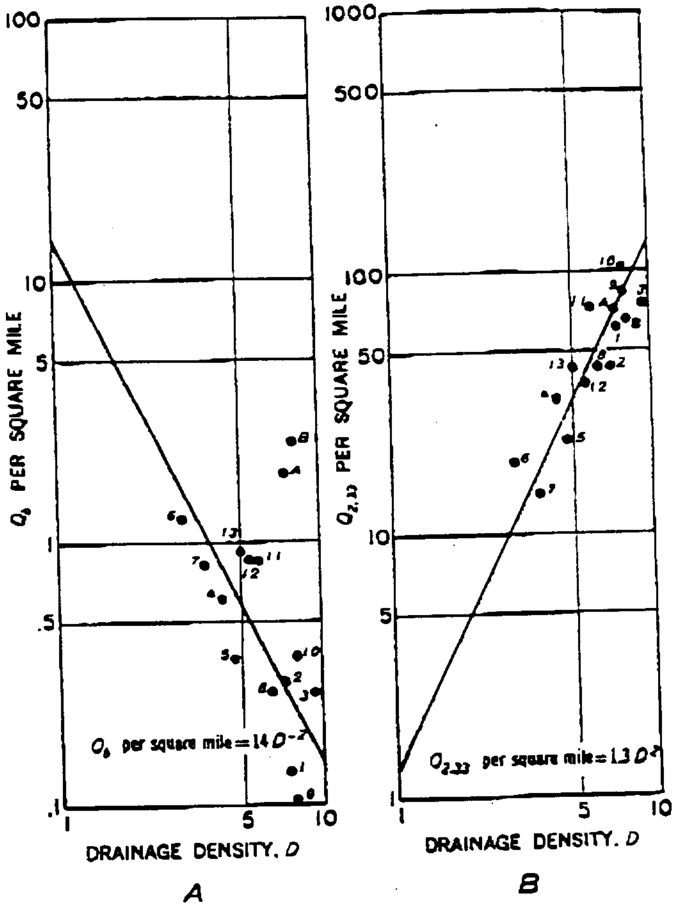

FIGURE 3.—The relation of drainage density to base flow and floods. *A*, Base flow (Q_b). *B*, Mean annual flood ($Q_{2.33}$).

The relation of drainage density to the 10-day base flow recession coefficient (r_{10}) was also investigated. It was found that this relation could be expressed quantitatively by the equation: Log $r_{10} = -0.008D^2$.

THE RELATION OF THE MEAN ANNUAL FLOOD TO DRAINAGE DENSITY

In the preceding discussion of the theoretical relation of drainage density to flood runoff it was concluded that flood runoff should vary directly with the second power of drainage density. Flood runoff for the basins studied was computed in terms of the mean annual flood per square mile ($Q_{2.33}$ per mi^2). A regression line having a slope of D^2 was plotted on this graph and the best fit gave an intercept with unit drainage density of about 1.3. This gives the equation:

$$Q_{2.33} \text{ per mi}^2 = 1.3D^2.$$

The departures from this mean trend line were compared graphically with average annual precipitation and average maximum rainfall intensities as given by the U.S. Weather Bureau. There was no apparent correlation. Graphical correlation was also made between mean annual flood and total relief. local relief, the ruggedness number (the product of relief and drainage density), stream slope, and valley-side slope. If significant correlation of flood peaks with these parameters exists, it is not apparent in the graphical plots.

The apparent lack of correlation between rainfall amount or intensity and flood intensity and between relief and flood intensity may be briefly illustrated by reference to points plotted in figure 3*B*. Basins A and B are located in the region of highest mean annual rainfall and rainfall intensity in the eastern U.S. In addition, the relief (1,500 ft) in these basins places them among those with the highest relief in the total sample, yet they plot on and below the average for the basins. Basin 10 has the highest mean annual flood (102 cfs per mi^2), but its relief is only 200 feet. Basin 13 has a relief of 2,600 feet and a high average annual precipitation (49 in.), but departure of its mean flood intensity from the average is not significantly high.

Hydrologists have found that the delay time (and hence the attenuation) of flood peaks is composed of two parts; the inlet or overland-flow time, and the channel-transit time. The present study deals with inlet times of monolithologic terrancs. The writer's analysis of flood runoff per unit area at gaging stations in the Appalachian Plateaus of eastern Kentucky (a basically monolithologic terrane) suggests that inlet time is dominant over channel transit time up to about 75 to 100 square miles in drainage area; however, because the channel transit time increases continuously, for larger basins it tends to become the dominant component of the flood-peak lag.

CONCLUSIONS

This paper has presented evidence that drainage density, surface-water runoff, and the movement of ground water are parts of a single hydrologic system controlled by the transmissibility of the bedrock and its overlying soil mantle. A mathematical model of such a system, constructed by Jacob (1943; 1944), has been adapted to show that transmissibility (T) is related to ground-water recharge (W), to drainage density (D) and to the height of the water table at the water table divide (h_0). The equation for this relation is:

$$T=\frac{WD^{-2}}{8h_0}.$$

If W and h_0 are constant, the equation may be simplified to $T=KD^{-2}$. Inasmuch as the rate of base flow (Q_b) or ground-water discharge into streams varies with and is controlled by transmissibility, $Q_b \propto D^{-2}$. A total of 13 small, basically monolithologic stream basins were selected in the eastern United States where rainfall and temperature are such that recharge can be considered to be a constant. It was found that the relation of base flow of the 13 streams to drainage density can be expressed by the equation Q_b per $mi^2 = 14D^{-2}$. The observed relation therefore is the same as that predicted by the Jacob model. Two stream basins in the southern Blue Ridge province, where rainfall is much higher than that of the 13 basins, have much higher base flows than comparable streams in the lower rainfall region.

Flood runoff as measured by mean annual flood ($Q_{2.33}$) was found to be closely related to drainage density; the equation may be expressed as $Q_{2.33}$ per $mi^2 = 1.3D^2$. It is concluded that the terrane transmissibility controls the amount of precipitation which passes through the underground system. The rejected or surface-water component increases with decreasing transmissibility. As surface-water runoff increases, an increase in the proliferation of stream channels is required for efficient removal of the runoff. The close relation of drainage density to mean annual flood per unit area indicates that the drainage network is adjusted to the mean flood runoff. Among the 15 basins in which flood runoff was correlated with drainage density, there are large and significant differences in relief, in valley-side and stream slopes, and in amounts and intensities of precipitation. These factors, however, have no discernible effect on the relation of the magnitude of the floods to drainage density. Transmissibility of the terrane appears to be the dominant factor in controlling the scale of drainage density and the magnitude of the mean annual flood for basins up to 75 to 100 square miles in area.

The research reported here should be of interest to geomorphologists in that it provides a quantitative physical model for the origin of one of the most important elements of landform characteristics, drainage density. In addition, it appears that surface-water hydrologists and ground-water hydrologists have far more in common in their hydrologic studies than is generally realized. Transmissibility or permeability is the most important aquifer characteristic in ground-water studies. The results reported in this paper indicate that the transmissibility or permeability of terranes drained by streams is also important in the study of surface-water hydrology. It is hoped that this study provides a theoretical physical basis for interdisciplinary hydrologic studies.

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